UNIT 6
QUINE-MCCLUSKEY METHOD

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Two-Level Logic Minimization

- Karnaugh map
  - Effective as # of variables < 5
- Quine-McCluskey method
  - Exact
- Espresso
  - Heuristic

Quine-McCluskey Method

- Contents
  - Determination of prime implicants
  - The prime implicant chart
- Reading
  - Sections 6.1 & 6.2

Two-Level Logic Minimization

Input: minterm expansion
Output: a minimum SOP
Procedure:
1. Generate all PIs
   - Make as many 1’s as possible in a circle
     - $XY' + XY = X$
2. Find the minimum SOP
   - Select a minimum number of circles to cover all 1’s as well as select large circles if possible

QM

Input: minterm expansion
Output: a minimum SOP
Procedure:
1. Generate all PIs
   - Start with minterms
   - Eliminate as many literals as possible from each term
     - $XY' + XY = X$
2. Find the minimum SOP
   - Find a min SOP in optimum way

Cost = (#terms, #literals)
Find all PIs

- K-map: paper-and-pencil
  - Circle 1’s
  - Apply $XY' + XY = X$

\[
f(a, b, c, d) = \Sigma m(0, 1, 2, 5, 6, 7, 8, 9, 10, 14)
\]

Find all PIs

- QM: Tabular
  1. Start with all minterms
  2. Group pairs of adjacent minterms
  3. Repeat grouping until no more grouping possible; mark (✓) + remove all covered terms.

\[
f(a, b, c, d) = \Sigma m(0, 1, 2, 5, 6, 7, 8, 9, 10, 14)
\]
Find a min SOP

   - Select fewest circles to cover all 1's
     - Select essential PI first
     - Cover the rest of 1's

\[ f(a, b, c, d) = \sum m(0, 1, 2, 5, 6, 7, 8, 9, 10, 14) \]

\[ f = a'c'd + a'bd + a'bc + b'c' + b'd' + cd' \]

QM: Tabular

1. Build the prime implicant chart (table)
2. Simplify the table using essential
   - Select an essential; delete covered minterms; repeat
3. Optimally select PIs by column covering

\[ f = a'c'd + a'bd + a'bc + b'c' + b'd' + cd' \]

Boolean matrix
Find a min SOP
QM: Tabular
1. Build the prime implicant chart (table)
2. Simplify the table using essential
   Select an essential; delete covered minterms; repeat
3. Optimally select PIs by column covering

\[ f(a, b, c, d) = \Sigma m(0, 1, 2, 5, 6, 7, 8, 9, 10, 14) \]

**Difficulty**

- Q: Column covering is HARD?!
- A: Yes
  - Consider a Boolean expression with \( n \) variables, in general
    - \( \sim 2^n \) minterms
    - \( \sim 3^n/n \) primes

**Espresso**:
- Speed up QM
- Generate some PIs
- Find min cover; repeat