Transmission Lines

1 Introduction

When the source radiates in a wide area, the energy spreads out. The radiated energy is not guided and the transmission of energy through radiation is inefficient. Directive antenna would have huge dimensions in the order of the wave length of the broadcasting electromagnetic waves.

We study the transverse electromagnetic (TEM) waves guided by transmission lines. The characteristics of TEM waves guided by transmission lines are the same as those for a uniform plane wave propagating in an unbounded dielectric medium.

The three most common types of guiding structures:
1. Parallel-plate transmission line. At microwave frequencies, parallel-plate transmission lines can be fabricated on a dielectric substrate using print-circuit technology. They are often called striplines.
2. Two-wire transmission line: power and telephone lines.
3. Coaxial transmission line: TV cables and the input cables to high-frequency precision measuring instruments.

2 Parallel-Plate Transmission Line (Geometric Model)

Physical Model

\[ \nabla^2 \mathbf{E} - \frac{1}{c^2} \frac{\partial^2 \mathbf{E}}{\partial t^2} = 0 \quad \text{since} \quad \mathbf{E} \propto e^{i\omega t} \rightarrow \nabla^2 \mathbf{E} + \frac{\omega^2}{c^2} \mathbf{E} = 0 \left( \nabla^2 \mathbf{E} + k_0^2 \mathbf{E} = 0 \right) \]

Assume it’s a plane wave propagate in the z with polarization in y direction.

\[ \frac{d^2}{dz^2} E_y + k_0^2 E_y = 0 \rightarrow \mathbf{E} = y\tilde{E}_0 e^{-ikz+i\omega t} \quad (+z), \quad \mathbf{E} = y\tilde{E}_0 e^{ikz+i\omega t} \quad (-z) \]

\[ \mathbf{E} = y\tilde{E}_0 e^{-ikz+i\omega t}, \quad \mathbf{H} = (\hat{\mathbf{z}} \times \hat{\mathbf{y}}) \frac{1}{\mu \nu} \tilde{E}_0 e^{-ikz+i\omega t} \]

\[ \mathbf{H} = -\hat{x} \frac{1}{\sqrt{\mu \varepsilon}} \tilde{E}_0 e^{-ikz+i\omega t} = H_x \hat{x} \quad \left( k = \frac{\omega}{v} = \omega \sqrt{\mu \varepsilon} \right) \]

When the TEM wave propagates in the parallel-plate transmission line, charge and current may be induced in the plates.

Crossing the boundary from dielectric medium to the perfect conduction plates:

\[ \nabla \times \mathbf{E} = 0 \rightarrow E_{/D} = E_{/C} = 0, \quad \nabla \cdot \mathbf{H} = 0 \quad H_{/D} = H_{/C} = 0 \]
at \( y = 0 \) (lower plate) \( \hat{n} = \hat{y} \), \( \nabla \cdot \vec{D} = \rho \)

\[ \Rightarrow E_{\perp c} = 0 \quad \& \quad \varepsilon \vec{E}_{\perp D} \cdot \hat{n} = \sigma_L \]

\[ \vec{E} = \hat{y} \vec{E}_0 e^{-ikz + i\omega t} \quad \Rightarrow \quad \vec{E}_L \cdot \hat{y} = \varepsilon \vec{E}_0 e^{-ikz + i\omega t} \]

\[ \nabla \times \vec{H} = \vec{J} \quad \Rightarrow \quad H_D w - H_c w = K_L w \quad \Rightarrow \quad K_L = \frac{1}{\mu} \frac{\vec{E}_0 e^{-ikz + i\omega t}}{\varepsilon} \]

**Use an Ampere’s loop to obtain the surface current.**

at \( y = d \) (upper plate) \( \hat{n} = -\hat{y} \)

\[ \sigma_U = -\varepsilon \vec{E}_0 e^{-ikz + i\omega t} = -\sigma_L \quad \& \quad \vec{K}_U = -\hat{z} \frac{1}{\mu} \frac{\vec{E}_0 e^{-ikz + i\omega t}}{\varepsilon} = \hat{z} H_x = -\vec{K}_L \]

In the dielectric media, the electric and magnetic fields satisfy the Maxwell’s eqs:

\[ \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad \& \quad \nabla \times \vec{B} = \mu_0 \varepsilon \frac{\partial \vec{E}}{\partial t} \]

\[ \Rightarrow \ nabla \times \vec{E} = -i\omega \mu \vec{H} \quad \& \quad \nabla \times \vec{H} = i\omega \varepsilon \vec{E} \]

\[ \vec{E} = \hat{y} \vec{E}_z (z,t), \quad \vec{H} = \hat{z} \vec{H}_x (z,t) \quad (\vec{E} \rightarrow V, \vec{H} \rightarrow \sigma \rightarrow I) \]

**Use Maxwell’s equation to get the two basic differential equations.**

\[ \begin{vmatrix} i \frac{\partial}{\partial x} & j \frac{\partial}{\partial y} & k \frac{\partial}{\partial z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & \varepsilon \frac{\partial}{\partial z} & 0 \end{vmatrix} = -i\omega \mu H_x \quad \Rightarrow \quad \frac{dE_x}{dz} = i\omega \mu H_x \quad \& \quad \frac{dH_x}{dz} = i\omega \varepsilon E_y \]

\[ \int_0^d \frac{dE_x}{dz} dy = i \omega \mu \int_0^d H_x dy, \quad \text{since} \quad \vec{K}_U = \hat{z} H_D \quad \Rightarrow \quad H_x = K_U \]
\[ -\frac{dV(z)}{dz} = i\omega\mu K_u(z) d = i\omega\mu \frac{d}{w}(K_u(z)) \omega = i\omega LI(z) \rightarrow -\frac{dV(z)}{dz} = i\omega LI(z) \]

\[ I(z) = K_u(z) w \quad \& \quad L = \mu \frac{d}{w} \text{ is the inductance per unit length } \left( -\frac{dV}{dz} \sim \frac{V}{L} \sim \frac{L}{L} \frac{dI}{dt} \right) \]

Use integration to obtain the voltage and current relation, and to calculate the inductance per unit length from our physical model.

\[ \int_0^w \frac{dH}{dz} \, dx = i\omega \varepsilon \int_0^w E_y \, dx \]

\[ \frac{d}{dz}(H, w) = \frac{d}{dz}(K_v, w) = \frac{d}{dz} I(z) = i\omega\varepsilon E_y, w \]

\[ = -i\omega\varepsilon \frac{w}{d} (-E_y, d) = -i\omega\varepsilon \frac{w}{d} V(z) = -i\omega CV(z) \]

\[ -\frac{dI(z)}{dz} = i\omega CV(z) \text{ where } C = \varepsilon \frac{w}{d} \text{ is the capacitance per unit length} \]

\[ \frac{I}{l} \sim \frac{1}{l} \frac{dQ}{dt} \sim \frac{1}{l} \frac{d}{dt} (\varepsilon V) \sim \frac{C}{l} \frac{dV}{dt} \]

Use integration to obtain the capacitance per unit length.

Time harmonic transmission line equations:

\[ -\frac{dV(z)}{dz} = i\omega LI(z) \quad \& \quad -\frac{dI(z)}{dz} = i\omega CV(z) \]

\[ \Rightarrow \frac{d^2V(z)}{dz^2} = -\omega^2 LCV(z) \quad \& \quad \frac{d^2I(z)}{dz^2} = -\omega^2 LCI(z) \]

\[ V(z) = V_0 e^{-ikz} \quad k = \omega \sqrt{LC} = \frac{\omega}{\sqrt{\varepsilon \mu \omega}} \quad V(z,t) = V_0 e^{-ikz + i\omega t} \]

\[ I(z) = I_0 e^{-ikz} \quad I(z,t) = I_0 e^{-ikz + i\omega t} \]

Use the derived differential equations to obtain the current and voltage variation as a function of \( z \) and \( t \).

The impedance:

\[ Z_0 = \frac{V(z)}{I(z)} = \frac{V_0}{I_0} = \frac{\omega LI_0}{kI_0} = \sqrt{\frac{L}{C}} = \sqrt{\frac{\mu d}{w}} \sqrt{\varepsilon} \quad \text{(not divided by length)} \]

The velocity of propagation along the line is:
Obtain impedance and speed of propagation waves.

**Microstrip Lines**
The development of solid-state microwave devices and systems has led to the widespread use of a form of parallel-plate transmission lines called microstrip lines or simply striplines.

The microstrip lines are closer to the parallel-plate transmission lines if \( w \gg d \).

**Lossy Parallel-Plate Transmission Lines**

\[
C = \frac{Q}{V} = \frac{\int \tilde{D} \cdot d\tilde{s}}{-\int \tilde{E} \cdot dl} = \frac{\int \tilde{E} \cdot d\tilde{s}}{-\int \tilde{E} \cdot dl}
\]

\[
R = \frac{V}{I} = \frac{-\int \tilde{E} \cdot d\tilde{l}}{\int \sigma \tilde{E} \cdot d\tilde{s}} = \frac{-\int \tilde{E} \cdot d\tilde{l}}{\int \sigma \tilde{E} \cdot d\tilde{s}} \Rightarrow RC = \frac{\varepsilon}{\sigma} \Rightarrow C = \frac{\varepsilon}{G} \frac{\sigma}{\varepsilon}
\]

Get surface impedance and resistance from the concept of power loss.

When the capacitance between the two conductors, the permittivity and the conductivity are known, we have

\[
\frac{G}{C} = \frac{\sigma}{\varepsilon} \quad (C = \varepsilon \frac{w}{d}) \Rightarrow G = \sigma \frac{w}{d}
\]

Averaged Power Dissipation: \( p_{\text{avg}} = -\hat{y}p_{\text{surface}} = \frac{1}{2} \text{Re}\{\hat{z}E_z \times \hat{x}H_x\} \) (Poynting’s theorem)

Upper plate: \( K_U = H_x \)

Define a surface impedance: \( Z_{\text{Surface}} = \frac{E}{K_{\text{Surface}}} = \frac{E_z}{H_x} \)

\( Z_{\text{Surface}} = R_{\text{Surface}} + iX_{\text{Surface}} \)
Obtain the surface impedance from the concept of attenuated waves in a perfect conductor.

A good conductor is a medium for which \( \sigma >> \omega \varepsilon \).

\[
\mathbf{E}(z,t) = \mathbf{E}_0 e^{(-iz+\omega t)}, \quad \frac{\partial^2}{\partial z^2} \mathbf{E} = \mu \varepsilon \frac{\partial \mathbf{E}}{\partial t} + \mu \varepsilon \frac{\partial^2 \mathbf{E}}{\partial t^2} \to
\]

\[-k^2 = i\mu \sigma \omega - \mu \varepsilon \omega^2 \sim i\mu \sigma \omega \]

\[
\frac{E}{H} = \frac{k}{k_c} = \frac{\omega \mu}{k_c} - \frac{\omega \mu}{\sqrt{-i\mu \varepsilon \omega}} = \sqrt{\frac{\mu \omega}{\sigma_c}} \frac{i}{\sqrt{2}} = \sqrt{\frac{\mu \omega}{\sigma_c}} \frac{1}{\sqrt{2}} + i \frac{1}{\sqrt{2}} = \sqrt{\frac{\mu \omega}{2\sigma_c}} (1+i)
\]

\[ Z_{\text{Surface}} = R_{\text{Surface}} + iX_{\text{Surface}} = \frac{E_z}{K_U} = \frac{E_z}{H_x} = \sqrt{\frac{\mu \omega}{2\sigma_c}} (1+i) \]

Obtain resistance of a parallel-plate transmission line.

\[ p_{\text{Surface}} = \frac{1}{2} \text{Re}\{K_U^2 Z_{\text{Surface}}\} = \frac{1}{2} K_U^2 R_{\text{Surface}}, \quad P = \omega p_{\text{Surface}} = \frac{1}{2} \omega K_U^2 R_{\text{Surface}} = \frac{1}{2} \frac{P}{w} \left( \frac{R_{\text{Surface}}}{w} \right) \]

The effective series resistance per unit length for both plates of a parallel-plate transmission line of width \( w \) is

\[ R = 2 \left( \frac{R_{\text{Surface}}}{w} \right) = \frac{2}{w} \sqrt{\frac{\mu \omega}{2\sigma_c}} \]  
(two times from the viewpoint of power dissipation) (per unit length)

\[ Z_{\text{Surface}} = \frac{E}{K} = \frac{E_l w}{K w l} = \frac{V_w}{l} = \frac{R w}{l} \]  
(not divided by length)
3 General Transmission-Line Equations

(Electronic Circuit Model)

\[
C = \frac{Q}{V} = \frac{\oint \vec{D} \cdot d\vec{s}}{\oint \vec{E} \cdot d\vec{l}} = \frac{\oint \vec{\varepsilon} \vec{E} \cdot d\vec{s}}{\oint \vec{E} \cdot d\vec{l}}
\]

\[
R = \frac{V}{I} = \frac{\oint \vec{E} \cdot d\vec{l}}{\oint \vec{J} \cdot d\vec{s}} = \frac{\oint \sigma \vec{E} \cdot d\vec{s}}{\oint \sigma \vec{E} \cdot d\vec{s}} \Rightarrow RC = \frac{\varepsilon}{\sigma} \Rightarrow \frac{C}{G} = \frac{\varepsilon}{\sigma}
\]

The capacitor is the capability of storing charges per volt. The conductor is the current flow per volt.

\[
G = C \frac{\sigma}{\varepsilon} \quad (C = \varepsilon \frac{w}{d} \text{ in parallel plate transmission lines}) \Rightarrow G = \sigma \frac{w}{d}
\]

\[
v(z,t) - R\Delta z i(z,t) - L\Delta z \frac{\partial i(z,t)}{\partial t} = v(z + \Delta z,t)
\]

\[- \frac{\partial v(z,t)}{\partial z} = Ri(z,t) + L \frac{\partial i(z,t)}{\partial t}
\]

\[
i(z,t) - G\Delta z v(z,t) - C\Delta z \frac{\partial v(z,t)}{\partial t} = i(z + \Delta z,t)
\]

\[- \frac{\partial i(z,t)}{\partial z} = Gv(z,t) + C \frac{\partial v(z,t)}{\partial t}
\]

let \( v(z,t) = \text{Re}\{V(z)e^{i\omega t}\} \) and \( i(z,t) = \text{Re}\{I(z)e^{i\omega t}\} \)

\[
\Rightarrow - \frac{dV(z)}{dz} = (R + i\omega L)I(z) \quad \text{and} \quad - \frac{dI(z)}{dz} = (G + i\omega C)V(z)
\]

\[
\frac{d^2V}{dz^2} = \frac{d}{dz}(-(R + i\omega L)I) = (R + i\omega L)\left(- \frac{dI}{dz}\right) = (R + i\omega L)(G + i\omega C)\frac{dV(z)}{dz}
\]

let \( V(z) = e^{-kz} \) \& \( \frac{d^2V}{dz^2} = k^2V(z) \Rightarrow k^2 = (R + i\omega L)(G + i\omega C) \Rightarrow \sqrt{(R + i\omega L)(G + i\omega C)}
\]
let  \( k = \alpha + i\beta = \sqrt{R + i\omega L} (G + i\omega C) \) (\( \alpha \): attenuation constant,  \( \beta \): phase constant)

\[
V(z) = V_0^+ e^{-kz} + V_0^- e^{kz} \quad \text{and} \quad I(z) = I_0^+ e^{-kz} + I_0^- e^{kz} \quad (v(z,t) = V_0^+ e^{-kz + i\omega t} + V_0^- e^{kz + i\omega t} \quad \text{and} \quad i(z,t) = I_0^+ e^{-kz + i\omega t} + I_0^- e^{kz + i\omega t})
\]

\[
- \frac{dV(z)}{dz} = (R + i\omega L)I(z) \rightarrow kV_0^+ e^{-kz} - kV_0^- e^{kz} = (R + i\omega L)(I_0^+ e^{-kz} + I_0^- e^{kz})
\]

\[
\frac{V_0^+}{I_0} = \frac{V_0^-}{I_0} = \frac{R + i\omega L}{k}
\]

characteristic impedance:  \( Z_0 = \frac{R + i\omega L}{k} = \sqrt{\frac{R + i\omega L}{G + i\omega C} (\propto \frac{V}{I} \frac{l_z}{l_z})} \) (not divided by length)

Example: Demonstrate the analog between the wave characteristics on a transmission line and uniform plane waves in a lossy medium.

In a lossy medium, we apply the model used to describe frequency dependent permittivity. The permittivity is therefore a complex number,  \( \varepsilon = \varepsilon' - i\varepsilon'' \). The same as permittivity, we may have complex permeability,  \( \mu = \mu' - i\mu'' \).

The Maxwell’s equations to relative magnetic and electric fields are:

\[
\nabla \times \vec{E} = -\mu \frac{\partial \vec{H}}{\partial t} = -i\omega (\mu' - i\mu'') \vec{H} \quad \& \quad \nabla \times \vec{H} = \varepsilon \frac{\partial \vec{E}}{\partial t} = i\omega (\varepsilon' - i\varepsilon'') \vec{E}
\]

Assume a uniform plane wave characterized by an  \( E_x \) that varies with only with  \( z \).
(Keep in mind that a wave function propagating in the  \( z \) direction with a polarization in the  \( x \) direction.)

\[
- \frac{\partial E_x(z)}{\partial z} = i\omega (\mu' - i\mu'') H_y = (\omega \mu'' + i\omega \mu') H_y
\]

\[
\begin{vmatrix}
\hat{i} & \hat{j} & \hat{k} \\
\frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\
0 & H_y & 0
\end{vmatrix} = - \frac{\partial H_z}{\partial z} \hat{i} = i\omega (\varepsilon' - i\varepsilon'') E_x \hat{j} \rightarrow - \frac{\partial H_y}{\partial z} = (\omega \varepsilon'' + i\omega \varepsilon') E_x
\]

\[
\frac{\partial E_y(z)}{\partial z} = - \frac{\partial}{\partial z} (\omega \mu'' + i\omega \mu') H_y = (\omega \mu'' + i\omega \mu') (\omega \varepsilon'' + i\omega \varepsilon') E_x(z)
\]

let  \( E_x(z) = E_{x0} e^{-kz} \quad \& \quad k_z^2 = (\omega \mu'' + i\omega \mu') (\omega \varepsilon'' + i\omega \varepsilon') \rightarrow \frac{\partial^2 E_x}{\partial z^2} = k_z^2 E_x \quad \& \quad \frac{\partial^2 H_y}{\partial z^2} = k_z^2 H_y
\]
Derived from general transmission line equations, we have

\[ k = \sqrt{(R + i\omega L)(G + i\omega C)} \]

Three limiting cases:

1. **Lossless Line** \((R = 0, G = 0)\). There is no real part in \(k\).

   (a) Propagation constant: \(k = i\omega \sqrt{LC}, \quad \alpha = 0, \quad \beta = \omega \sqrt{LC}\)

   (b) Phase velocity: \(v_{\text{phase}} = \frac{\omega}{\beta} = \frac{1}{\sqrt{LC}}\)

   (c) Characteristic impedance: \(Z_0 = R_0 + iX_0 = \frac{R + i\omega L}{G + i\omega C} = \sqrt{\frac{L}{C}}, \quad R_0 = \frac{L}{C}\), \(X_0 = 0\)

2. **Low-Loss Line** \((R \ll \omega L, G \ll \omega C)\)

   (a) Propagation constant:

   \[
   k = i\omega \sqrt{LC} \left(1 - i \frac{R}{\omega L} \right) \left(1 - i \frac{G}{\omega C} \right) = i\omega \sqrt{LC} \left(1 - i \frac{R}{2\omega L} - i \frac{G}{2\omega C}\right)
   \]

   \[
   = R \left[ \frac{C}{2} + G \frac{L}{2C} \right] + i\omega \sqrt{LC}
   \]

   \[
   \alpha \approx R \left[ \frac{C}{2} + G \frac{L}{2C} \right], \quad \beta \approx \omega \sqrt{LC}
   \]

   (b) Phase velocity: \(v_{\text{phase}} = \frac{\omega}{\beta} \approx \frac{1}{\sqrt{LC}}\)

   (c) Characteristic impedance:

   \[
   Z_0 = R_0 + iX_0 = \sqrt{\frac{R + i\omega L}{G + i\omega C}} \Rightarrow
   \]

   \[
   Z_0 = \sqrt{\frac{L}{C}} \left(1 - i \frac{R}{\omega L} \right)^{1/2} \left(1 - i \frac{G}{\omega C} \right)^{-1/2} \approx \sqrt{\frac{L}{C}} \left(1 - i \frac{R}{2\omega L} + i \frac{G}{2\omega C}\right), \quad R_0 = \frac{L}{\sqrt{C}},
   \]

   \[
   X_0 = \frac{G}{2\omega C} \sqrt{\frac{L}{C}} - \frac{R}{2\omega} \frac{1}{\sqrt{LC}}
   \]

3. **Distortionless Line** \((R / L = G / C)\)

   (a) Propagation constant:

   \[
   k = i\omega \sqrt{LC} \left(1 - i \frac{R}{\omega L} \right) = \sqrt{\frac{C}{L}} \left(R + i\omega \sqrt{LC}\right)
   \]

   \[
   \alpha \approx R \sqrt{\frac{C}{L}}, \quad \beta \approx \omega \sqrt{LC}
   \]
(b) Phase velocity: \( v_{\text{phase}} = \frac{\omega}{\beta} = \frac{1}{\sqrt{LC}} \)

(c) Characteristic impedance: \( Z_0 = R_0 + iX_0 = \sqrt{\frac{R + i\omega L}{G + i\omega C}} = \sqrt{\frac{R}{G}} = \sqrt{\frac{L}{C}} \rightarrow, \)

\( R_0 = \sqrt{\frac{L}{C}}, \quad X_0 = 0 \)

The different frequency components travel along a transmission line at the same velocity in order to avoid distortion. For a lossy line, wave amplitude will be attenuated, and **distortion will result when different frequency components attenuate differently**, even when they travel with the same velocity.

Example: It is found that the attenuation on a 50 (\( \Omega \)) distortionless transmission line is 0.01 (dB/m). The line has a capacitance of 0.1 (nF/m).

a) Find the resistance, inductance, and conductance per meter of the line.

b) Find the velocity of wave propagation.

c) Determine the percentage to which the amplitude of a voltage traveling wave decrease in 1 (km) and in 5 (km).

50 (\( \Omega \)) \( \rightarrow \) real part of characteristic impedance

1 neper = 8.6858896 decibels \( R / L = G / C \)

\( a) \quad R_0 = \sqrt{\frac{L}{C}} = 50\Omega, \quad \alpha = R \sqrt{\frac{C}{L}} = 0.01(dB/m) = \frac{0.01}{8.69}(NP/m) = 1.15 \times 10^{-3}(NP/m) \)

\( C = 10^{-10}(F/m) \Rightarrow L = R_0^2C = 2.5 \times 10^{-7}(H/m) = 0.25(\mu H/m) \)

\( R = \alpha R_0 = 1.15 \times 10^{-3} \times 50 = 0.057(\Omega/m) \)

\( G = \frac{RC}{L} = 22.8(\mu S/m) \)

b) \( v_{\text{phase}} = \frac{1}{\sqrt{LC}} = 2 \times 10^8 m/s \)

c) After 1 (km), \( V_2/V_1 = e^{-\alpha} = e^{-1.15 \times 10^{-3} \times 1000} = 0.317 \).

After 5 (km), \( V_2/V_1 = e^{-\alpha} = e^{-1.15 \times 10^{-3} \times 5000} = 0.0032 \).

**Transmission-Line Parameters**

The electrical properties of a transmission line are completely characterized by its four parameters R, L, G, and C.
\[ \text{Let } R = 0 \Rightarrow k = \sqrt{i \omega L (G + i \omega C)} = i \omega \sqrt{LC} \left(1 + \frac{G}{i \omega C}\right) \]

Compare with \( k^2 = \mu \sigma \omega - \mu \omega^2 \Rightarrow k = i \omega \sqrt{\mu \sigma} \left(1 + \frac{\sigma}{i \omega \varepsilon}\right) \)

\[ \Rightarrow LC = \mu \varepsilon \]

1. Two-wire transmission line. The wires have a radius \( a \) and are separated by a distance \( D \).

(Obtain the capacitance from the method of images.)

\[ C = \frac{\pi \varepsilon}{\cosh^{-1}(D/2a)} \quad (LC = \mu \varepsilon) \Rightarrow \quad L = \frac{\mu}{\pi} \cosh^{-1}\left(\frac{D}{2a}\right), \quad G = \frac{\pi \sigma}{\cosh^{-1}(D/2a)} \]

\[ P_{\text{surface}} = 2 \pi a P_{\text{surface}} = 2 \pi a \frac{1}{2} K^2 R_{\text{surface}} = \frac{1}{2} I^2 \left(\frac{R_{\text{surface}}}{2 \pi a}\right) \]

\[ R = 2 \left(\frac{R_s}{2 \pi a}\right) = \frac{\sqrt{\frac{\mu}{\varepsilon}}}{\pi a} \quad (\text{The factor of 2 is not from series resistance but from power dissipation.}) \]

2. Coaxial transmission line.

\[ L = \frac{\mu}{2 \pi} \ln \frac{b}{a} \quad (LC = \mu \varepsilon) \Rightarrow \quad C = \frac{2 \pi \varepsilon}{\ln(b/a)} \quad \text{and} \quad G = \frac{2 \pi \sigma}{\ln(b/a)} \]

\[ I = 2 \pi a K_s, \quad P_{s,1} = 2 \pi a P_{s,1} = \frac{1}{2} I^2 \left(\frac{R_s}{2 \pi a}\right) \quad \& \quad P_{s,0} = \frac{1}{2} I^2 \left(\frac{R_s}{2 \pi b}\right) \]

\[ R = \frac{R_s}{2 \pi} \left(\frac{1}{a} + \frac{1}{b}\right) = \sqrt{\frac{\mu}{2 \varepsilon}} \left(\frac{1}{a} + \frac{1}{b}\right) \]

**Attenuation Constant from Power Relations**

The attenuation constant of a traveling wave on a transmission line:

\[ \alpha = \text{Re} \{k\} = \text{Re} \{\sqrt{(R + i \omega L)(G + i \omega C)}\} \]

Find the attenuation constant from power relation:

\[ V(z) = V_0 e^{-\alpha z} \quad \& \quad I(z) = I_0 e^{-\alpha z} = \frac{V_0}{Z_0} e^{-\alpha z} \]

The time averaged power propagated along the transmission line is:

\[ P(z) = \frac{1}{2} \text{Re} \{I(z)V(z)\} = \frac{V_0^2}{2 |Z_0|^2} e^{-2 \alpha z} \]

Time-average power loss per unit length: \( P_L = -\frac{\Delta P}{\Delta z} = -\frac{dP}{dz} \)
Exercise: The following characteristics have been measured on a lossy transmission line at 100 MHz:

\[ Z_0 = 50 + i0 \quad (\Omega), \quad \alpha = 0.01 \text{ (dB/m)}, \quad \beta = 0.8\text{p (rad/m)} \]

Determine \( R, L, G, \) and \( C \) for the line.