Calculus 0314

Quiz 2.

(1) Determine whether the series is absolutely convergent, conditionally convergent or divergent.

(a) \(\sum_{n=1}^{\infty} \frac{(-1)^n n}{5 + n}\)  
(b) \(\sum_{n=1}^{\infty} \frac{\cos(\frac{n\pi}{2})}{n!}\)  
(c) \(\sum_{n=1}^{\infty} \frac{2 \cdot 4 \cdot 6 \cdots (2n)}{n!}\)  
(d) \(\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\sqrt{n}}\).

(a) divergent, (b) absolutely convergent, (c) divergent, (d) conditionally convergent.

(2) Find the radius of convergence and interval of convergence. (15%)

(a) \(\sum_{n=1}^{\infty} \frac{x^n}{\sqrt{n}}\)  
(b) \(\sum_{n=1}^{\infty} \frac{(-1)^n (x + 2)^n}{n2^n}\)  
(c) \(\sum_{n=1}^{\infty} \frac{(n!)^2}{(2n)!} x^n\).

(a) radius= 1, interval= [−1, 1]; (b) radius= 2, interval= (−4, 0]; (c) radius= 4, interval= (−4, 4).

(3) Suppose \(\sum_{n=0}^{\infty} c_n(x - 2)^n\) converges when \(x = 6\) and diverges when \(x = -4\). What can be said about the convergence or divergence of the following series? (8%)

(a) \(\sum_{n=1}^{\infty} c_n x^n\)  
(b) \(\sum_{n=1}^{\infty} (-1)^n c_n x^n\)  
(c) \(\sum_{n=0}^{\infty} c_n 7^n\)  
(d) \(\sum_{n=0}^{\infty} c_n 8^n\).

(4) Suppose the series \(\sum_{n=0}^{\infty} c_n x^n\) and \(\sum_{n=0}^{\infty} d_n x^n\) have, respectively, radius of convergence 2 and 3.

(a) What is the radius of \(\sum_{n=0}^{\infty} (c_n + d_n) x^n\)? (5%) 2.

(b) What is the radius of \(\sum_{n=0}^{\infty} c_n x^{2n}\)? (5%) \(\sqrt{2}\).

(5) Find a power series representation for the function and determine the radius of convergence.

(a) \(f(x) = \ln(5 - x)\)  
(b) \(f(x) = \frac{x^2}{(1 + x)^2}\)  
(c) \(f(x) = \ln 5 - \sum_{n=1}^{\infty} \frac{x^n}{n \cdot 5^n}\) and radius = 5.

(b) \(f(x) = \sum_{n=0}^{\infty} (-1)^n(n + 1)x^{n+2}\), and radius = 1.

(6) (a) Find \(\sum_{n=1}^{\infty} nx^{n-1}\) as \(|x| < 1\). (Hint: \((x^n)' = nx^{n-1}\).) (5%) \(\frac{1}{(1-x)^2}\).

(b) \(\sum_{n=1}^{\infty} nx^n =?\) as \(|x| < 1\). (5%) \(\frac{x}{(1-x)^2}\).

(c) \(\sum_{n=1}^{\infty} \frac{n}{2^n} =?\) (5%) 2.

(7) Find the Maclaurin series of \(f(x) = \cos x\). (Assume that \(f\) has a power series expansion). (6%)

\(f(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} x^{2n}\).

(8) Use the binomial series to expand the function \(f(x) = \frac{x}{\sqrt{4 + x^2}}\) and state its radius of convergence. (6%) \(f(x) = \frac{x}{2} + \sum_{n=1}^{\infty} (-1)^n \frac{1 \cdot 3 \cdots (2n - 1)}{2n+1 n!} x^{2n+1}\), and radius=2.

(9) (a) Find the Maclaurin series of \(f(x) = \sqrt{1 + x^2}\). (5%)

(b) Evaluate \(f'(10)\). (3%) Should be corrected by \(f'(10)(0)\).

(a) \(1 + \frac{x^2}{2} + \sum_{n=1}^{\infty} (-1)^{n-1} \frac{1 \cdot 3 \cdots (2n - 3)}{2^n n!} x^{2n}\), (b) \(f'(10) = \frac{-10}{\sqrt{101}}\), \(f'(10)(0) = 99, 225\).