Chapter 7 – Robot Control and Compliance

7.1 Introduction

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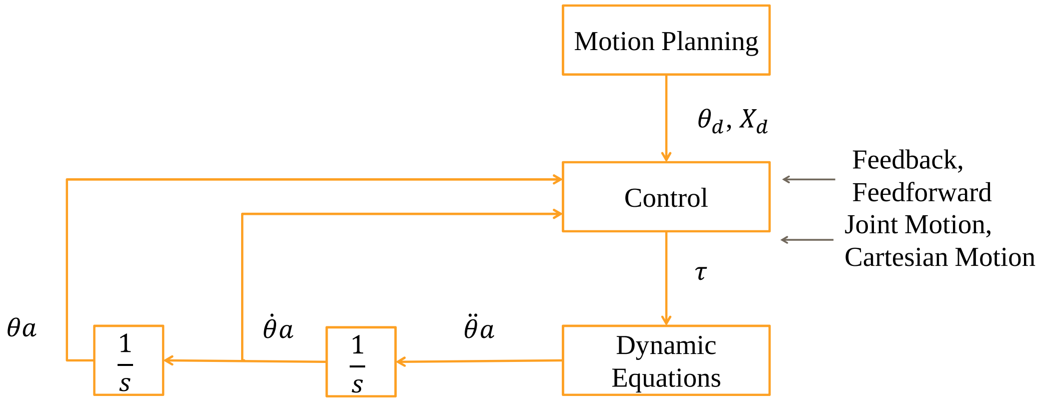
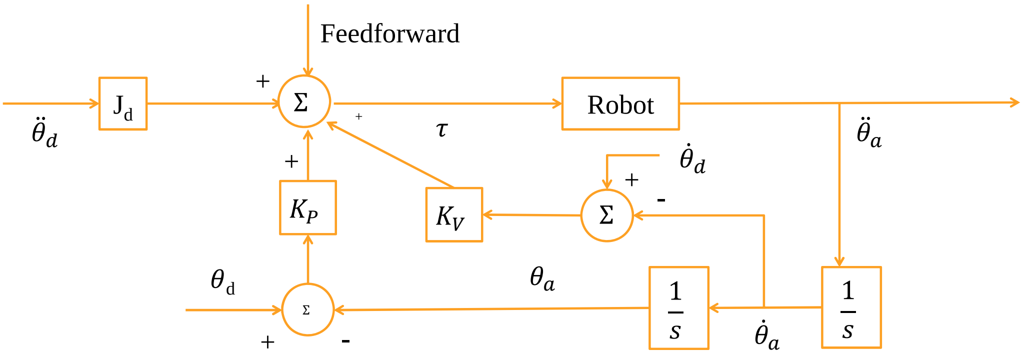
7.1 Introduction



7.1 Introduction



$$\tau = J_a\ddot{\theta}_a \qquad \tau = J_d\ddot{\theta}_d + K_p(\theta_d - \theta_a) + K_v(\dot{\theta}_d - \dot{\theta}_a)$$

$$J_d(\ddot{\theta}_d - \ddot{\theta}_a) = J\ddot{\theta}_e \qquad J_d\ddot{\theta}_e + K_v\dot{\theta}_e + K_p\theta_e = 0$$

$$W_n^2 = \frac{K_p}{J} \qquad \xi = 1 \qquad 2W_n = \frac{K_v}{J}$$

7.1 Introduction

- To prevent exciting structural oscillations due to the unmodeled flexibility, W_n is limited to less than 0.5 W_{res}, i.e. $W_n \leq \frac{1}{2}W_{res}$ (a rule of thumb).

- Typical industrial manipulators have structural resonances in the range of 5Hz to 25 Hz. If the robot manipulator is stiffer, the resonant frequency can be increased.

7.1 Introduction

- Individual Joint PID Control

$$\tau = K_v \dot{E} + K_p E + K_i \int E dt + \ddot{\theta}_d$$

  - With this control law, no decoupling is executed, the motion of each joint affects the other joints. These interactions causes errors which are suppressed by the error driven control law. The fixed gains for each joint will not critically damp the response for all configurations.

7.1 Introduction

- Proof of the stability of PD control law under regulation

$$\tau = M(\theta)\ddot{\theta} + V(\theta,\dot{\theta}) + G(\theta)$$

Control law

$$\tau = K_p E - K_d \dot{\theta} + G(\theta)$$

Lyapunov function

$$v = \frac{1}{2}\dot{\theta}^T M(\theta)\dot{\theta} + \frac{1}{2}E^T K_p E$$

$$\dot{v} = \frac{1}{2}\dot{\theta}^T \dot{M}(\theta)\dot{\theta} + \dot{\theta}^T M(\theta)\ddot{\theta} - E^T K_p \dot{\theta}$$

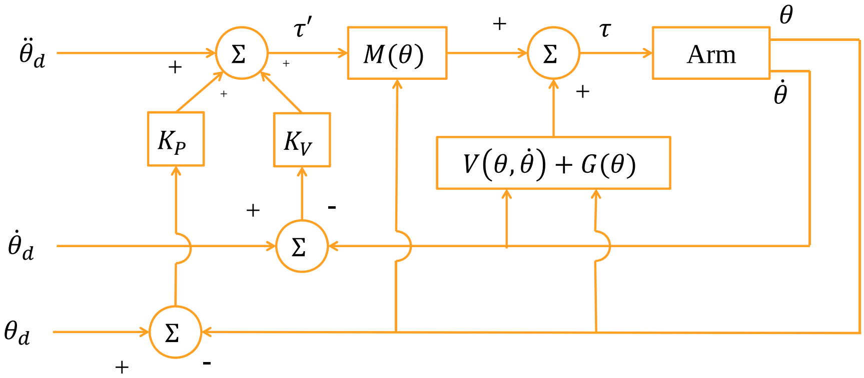
$$= \frac{1}{2}\dot{\theta}^T \dot{M}(\theta)\dot{\theta} - \dot{\theta}^T K_d \dot{\theta} - \dot{\theta}^T V(\theta,\dot{\theta})$$

$$\dot{v} = -\dot{\theta}^T K_d \dot{\theta} < 0$$

$$\frac{1}{2}\dot{\theta}^T \dot{M}(\theta)\dot{\theta} = \dot{\theta}^T V(\theta,\dot{\theta})$$

7.1 Introduction

- □ Computed-Torque Method



7.1 Introduction

- □ Dynamics $\tau = M(\theta)\ddot{\theta} + V(\theta,\dot{\theta}) + G(\theta)$
- □ Let control torque $\tau = \alpha\tau' + \beta$

$$\alpha = M(\theta), \beta = V(\theta,\dot{\theta}) + G(\theta)$$

$$\tau' = \ddot{\theta}_d + K_v\dot{E} + K_p E$$

$$E = \theta_d - \theta$$

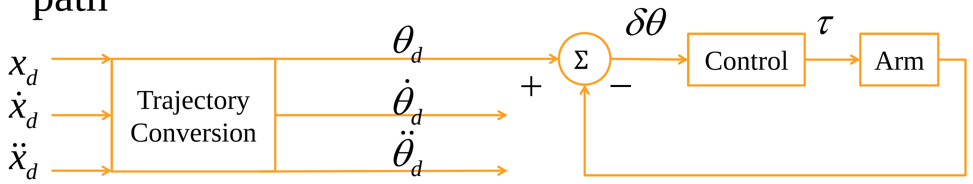
$$\ddot{E} + K_v\dot{E} + K_p E = 0$$

- □ Error equation $\ddot{E} + K_v\dot{E} + K_p E = 0$
- □ Let K_V and K_P be diagonal
- □ In some sense, a joint-by-joint, basis

$\ddot{e}_i + K_{vi}\dot{e} + K_{pi}e = 0$ For i=1, 2, …n (Decoupled system)

7.2 Cartesian-Based Control

- A joint-based control scheme to track a Cartesian path

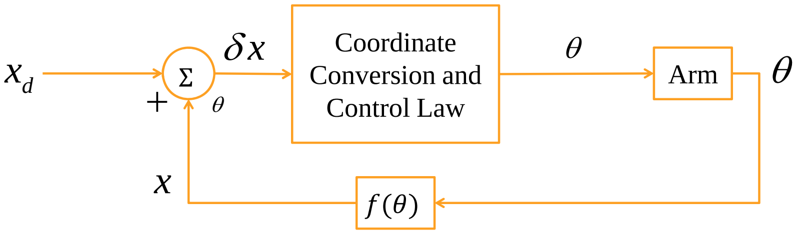


- It requires that:

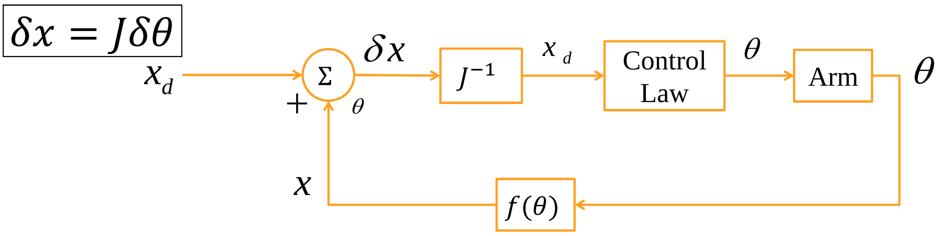
$$\theta_d = f^{-1}(x_d)$$
$$\dot{\theta}_d = J^{-1}(\theta)\dot{x}_d$$
$$\ddot{\theta}_d = \dot{J}^{-1}(\theta)\dot{x}_d + J^{-1}(\theta)\ddot{x}_d$$

7.2 Cartesian-Based Control

- A Cartesian-based Control Scheme



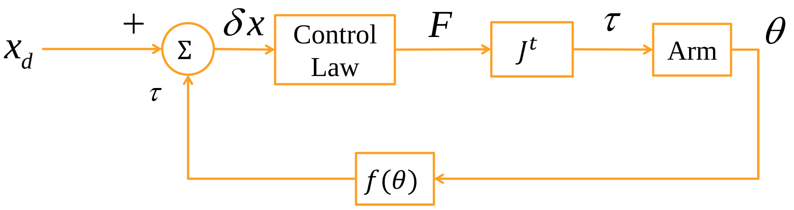
- An inverse Jacobian Cartesian Control Scheme



$$\boxed{\delta x = J\delta\theta}$$

7.2 Cartesian-Based Control

- ☐ An transpose Jacobian Cartesian control scheme



$$\begin{cases} \textit{Adaptive Control} \\ \textit{Robust Control} \end{cases}$$

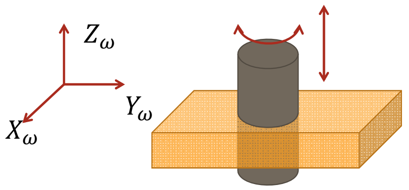
7.3 Compliance

- ☐ What is compliance? $dW = F \cdot dx \neq 0$ (energy transfer) (compliance motion)
- ☐ What are the compliance tasks?
  - ☐ Assembly, Deburring, Screw-driving, Drilling, Chipping, Grinding, etc.
- ☐ Passive compliance: Passive mechanical compliance built in the manipulator (special purpose)
- ☐ Active compliance: Implemented in the software control loop (programmable)
- ☐ Those that the position of a manipulator is constrained by the task

Ref: "Compliance and Force Control for Computer Controlled Manipulation" M.T. Masson, IEEE trans. Systems and Cybernetics Vol. 11(6), pp. 481-432, 1981

7.3 Compliance

- A compliant motion example: insertion of a peg into a hole



2 degree of freedom in translation and rotation

Move to D with

$$\begin{cases} Force\ X = 0 \\ Force\ Y = 0 \\ Torque\ X = 0 \\ Torque\ Y = 0 \end{cases}$$

Where D is the goal position, the peg is free to rotate and move in the Z-direction

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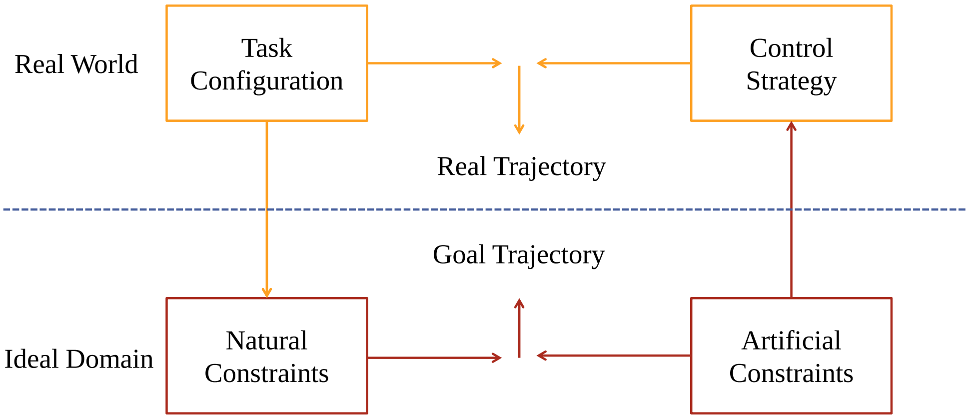
7.3 Compliance

- Overview of control strategy synthesis



Real World

Task Configuration

Control Strategy

Real Trajectory

Goal Trajectory

Ideal Domain

Natural Constraints

Artificial Constraints

7.3 Compliance

- Natural Constraints:
  - Task configurations which are represented by equations relating to the components of the manipulator’s force and velocity.
- Artificial Constraints:
  - Control strategies for the manipulator which is represented as Equation’s force and velocity.
- Goal Trajectory:
  - Desired position or force of the manipulator as a function of time.

7.3 Compliance

- Synthesis of Strategies
  - ① The task is modeled as a set of N.C. in the ideal domain.
  - ② The synthesis method is applied in the ideal domain to obtain a set of A.C.
  - ③ A.C. are transformed into the corresponding real-world control strategies.

7.3 Compliance

Example

$$\text{N.C.}\ \begin{cases} V_x = 0, W_x = 0 \\ V_y = 0, W_y = 0 \\ f_z = 0, g_z = 0 \end{cases} \qquad \text{A.C.}\ \begin{cases} f_x = 0, g_x = 0 \\ f_y = 0, g_y = 0 \\ V_z = K_1(t), W_z = K_2(t) \end{cases}$$

Let $\underline{v} = (v_x, v_y, v_z, w_x, w_y, w_z)^t$ $\quad \underline{f} = (f_x, f_y, f_z, g_x, g_y, g_z)^t$

Then

$A\underline{v} = 0$

$B\underline{f} = 0$

$$A = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \qquad B = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Where $(S_v)^{\perp} = S_f = \{V_z, W_z\}$ $\qquad (S_v)^{\perp} = S_f = \{f_x, f_y, g_x, g_y\}$

7.3 Compliance

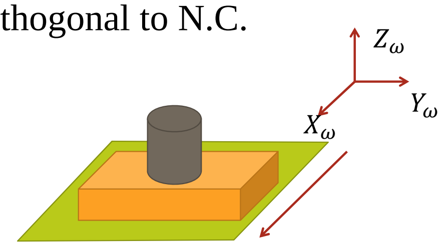
- In general A.C. are chosen to be orthogonal to N.C.

N.C.

$$\begin{cases} V_x = V_{belt}, W_x = 0 \\ V_y = 0, W_y = 0 \\ f_z = 0, g_z = 0 \end{cases}$$

A.C.

$$\begin{cases} f_x = 0, g_x = 0 \\ f_y = 0, g_y = 0 \\ V_z = K_1(t), W_z = K_2(t) \end{cases}$$



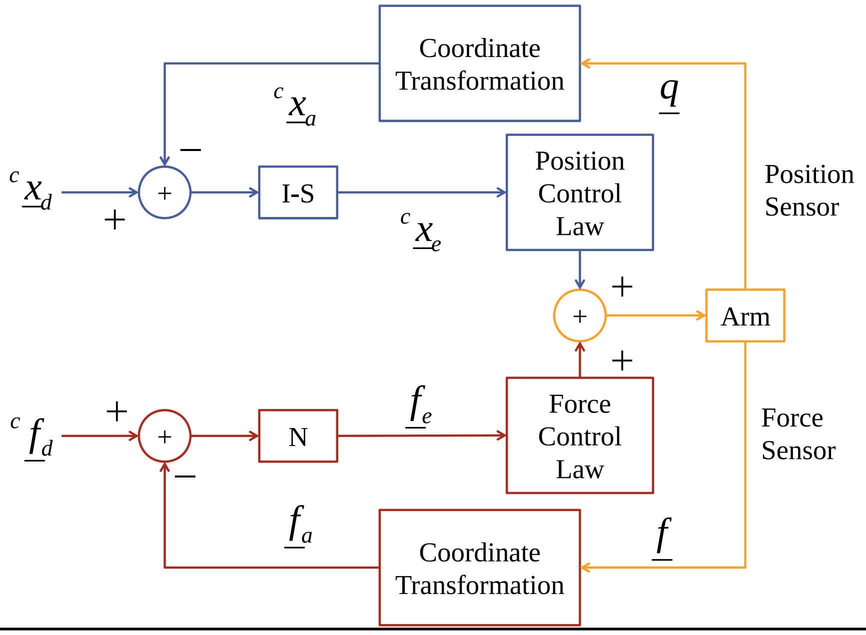
- Compare these two examples, the motion along the X-axis is taken by the force constraint along the X-axis. This constraint forces the peg to follow the hole whether stationary or moving.
- Example: How to describe the condition when a peg is resting at the bottom of a hole $N.C. \Rightarrow V_z = 0 \quad A.C. \Rightarrow f_z = \varepsilon$

7.4 Hybrid Position/Force Control

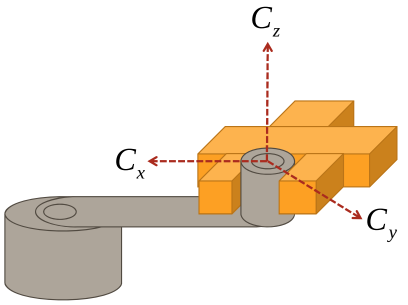
- Wrist sensor:
  - More sensitive, sophisticated in design, Cartesian information, easy to be damaged.
- Joint sensor:
  - Protected by the damping from the link, get joint information, not so sensitive, easy to design.

Ref: "Hybrid Position/Force Control of manipulators", M.H. Raibert and J.J. Craig. ASME J. Dynamic Systems, Measurement, and Control, Vol.102, PP.126-133, 1981

7.4 Hybrid Position/Force Control



7.4 Hybrid Position/Force Control



N.C.

$$\begin{cases} V_x = 0, & f_y = 0 \\ V_z = 0, & \tau_z = 0 \\ W_x = 0 & \\ W_y = 0 & \end{cases}$$

A.C.

$$\begin{cases} V_y = 0, & f_x = 0 \\ W_z = K_1(t), & f_z = 0 \\ & \tau_x = 0 \\ & \tau_y = 0 \end{cases}$$

7.4 Hybrid Position/Force Control

- Where: $\tau_i = \sum_{j=1}^{N} \left\{ \Gamma_{ij} \left[S_j \Delta f_j \right] + \psi_{ij} \left[1 - S_j \right] \Delta X_j \right\}$
  - τ_i: torque applied by the i^{th} actuator
  - Δf_j: force error in j^{th} DOF
  - Δx_j: position error in j^{th} DOF
  - Γ_{ij} *and* ψ_{ij}: force and position compensation respectively for the j^{th} input and i^{th} vector
  - S_j: Component of compliance selection vector
    - S_j: 1 DOF under force control
    - S_i: 1 DOF under position control
  - For example: if S={1, 0, 1, 1, 1, 0} then:

$$\tau_i = \psi_{i1}(\Delta f_1) + \psi_{i2}(\Delta f_2) + \psi_{i3}(\Delta f_3) + \psi_{i4}(\Delta f_4) + \psi_{i5}(\Delta f_5) + \psi_{i6}(\Delta f_6)$$

Note: S selection vector may vary as the task geometry and N.C. change

7.5 Impedance Control

- An approach to the control of dynamic interaction between a manipulator and its environment. The author proposed that control of position or force alone is inadequate; control of dynamic behavior is required.

Ref: "Impedance Control: An Approach to manipulator Part I – Theory, Part II - Implementation, Part III - Applications", N. Hogan, ASMF, J. Dynamic Systems, Measurement and Control, Vol.107, pp. 1-24, 1985

7.5 Impedance Control

- Analyze F=KX

$$(F = m\ddot{x} + B\dot{x} + Kx)$$

- Then the environment and the manipulator cannot be modeled separately.
- Physical System come in two types:
  - Admittance: accept "effort" inputs and yield "flow" outputs.
  - Impedance: accept "flow" inputs and yield "effort" outputs.
- For linear system, these two are interchangeable.
- For non-linear systems, these two are not always interchangeable.

7.5 Impedance Control

- Example:
  - When a manipulator is mechanically coupled to its environment, to ensure physical compatibility with the environment admittance, the manipulator should assume the behavior of an impedance. While a constrained inertial object can always be pushed on, it cannot always be moved.
- Remark:
  - ① Impedance can be summed, but admittance cannot.
  - ② It is not the impedance to be controlled. It is the position and orientation to be controlled to modulate the impedance.
  - ③ A controller for compliance tasks should be capable of modulating the impedance of the manipulator.

7.5 Impedance Control

- Implementation:
  - Define the desired equilibrium position X_0.

$$f_{\text{int}} = K(x_0 - x)$$

$$dx = J(\theta)d\theta, \ x = L(\theta)$$

$$\tau_{\text{int}} = J^t(\theta) f_{\text{int}}$$

  - Also consider the relation between force and velocity.

$$f_{\text{int}} = B(V_0 - V) \qquad \tau_{\text{int}} = J^t(\theta)B(V_0 - J(\theta)W)$$

$$V = J(\theta)W \qquad f_{\text{int}} = K(x_0 - x) + B(V_0 - V) - M\frac{dv}{dt} \quad (1)$$

  - The dynamics needs to be considered

$$\tau_{\text{int}} + \tau_{act} = H(\theta)\ddot{\theta} + V(\theta,\dot{\theta})\dot{\theta} + G(\theta) \quad (2)$$

  - From (1) and (2), the relation between f_{int} and τ_{act} can be solved,

$$(\tau_{act}, \theta, \dot{\theta}, \ddot{\theta})$$

7.5 Impedance Control

- Remark:
  - It can be seen that the input impedance varies along with the position and orientation. How to modulate it according to the task? Human do it with redundancy and coactivation.

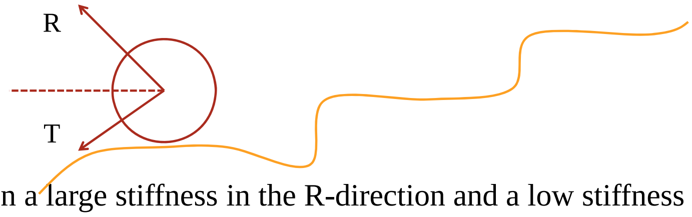
Ref:

① "An approach to sliding Mode-Based impedance Control", Z. Lu, S. Kawamura, and A. A. Goldenberg, IEEE Trans. Rob & Automation, Vol.11 (5), pp. 754-759, 1995

② "Hybrid Impedance Control of Robotic Manipulators", R. J. Anderson & M. W. Spong, IEEE J. Rob & Ant, vol. 4(5), pp, 549-556, 1998

③ "Robust Compliant Motion for Manipulators, Part I: The Fundamental Concepts of Compliant Motion", H. Kazerooni, T. B. Sheridan, and P. K. Hanpt, IEEE J. Rob & Aunt, vol. 2(2), pp. 83-91, 1986

7.5 Impedance Control



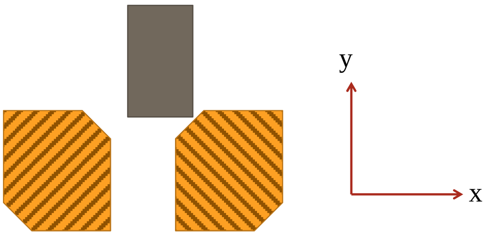
- To design a large stiffness in the R-direction and a low stiffness in the T-direction.
- Large stiffness in the R-direction causes the end-point to reject the external forces and stay close to the command trajectory. After stiffness in the R-direction is determined, the speed in the T-direction is determined by the volume of the bump. The larger the bump the slower the speed. If the stiffness in the T-direction is high.

① The cutting tool may stall.

② May cause the tool to move in the R-direction. Then, the surface is rougher when KB is smaller.

7.5 Impedance Control

- Example: Peg-in-Hole



  - A small stiffness in the X-direction to align itself with the axis of the hole. A large stiffness is chosen to reject the friction in the Y-direction.