An Pro-active Optimal Variable Speed Limit Control for Freeway local bottlenecks

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Content

- Research Background
- Dynamic Traffic Flow Modeling
- Reactive control Strategy
- Pro-active control Strategy
- Rolling Horizon operational Algorithm
Research Background

- Variable speed limit (VSL) control is one of the advanced traffic management strategies (ATMS) that has received increasing interest in the traffic community.

- A complete VSL system typically consists of a set of traffic sensors, several properly located VMS, and a real-time database as well as communication systems to convey information between all principal modules.

- The core VSL logic is to dynamically adjust the set of speed limits properly located along a target roadway segment so as to smooth the speed transition between the upstream free-flows and downstream congested traffic states, and thereby preventing the formation of excessive queues due to the shockwave impacts.
**System design features:**
The target freeway is subdivided into a number of segments. A set of detectors are installed along with VSLs to monitor and control the traffic speed. Each segment shall comprise only one on-ramp and one off-ramp for convenience of model formulations.
On each segment, the upstream detector is used to capture the arriving flow rate, and a downstream detector is designed to record the discharging rate from the bottleneck.

Also, additional detectors are placed at the on-ramps and off-ramps to record ramp arriving and departing flows.

VSL signs shall be installed between the upstream and downstream detectors.
Dynamic Traffic Flow Modeling

- For a segment $i$, the linear equations of macroscopic traffic flow models are:

$$d_i(k + 1) = d_i(k) + \frac{\Delta T}{\Delta l \ast n_i} \left[ q_{i-1}(k) - q_i(k) + r_i(k) - s_i(k) \right]$$

Where, $T$: Unit time interval; $\Delta l$: Length of subsection $i$; $n_i$: Number of lanes in subsection $i$;

$$q_i(k) = d_i(k)u_i(k)n_i$$

Mean traffic density  Transition flow rate  On-ramp flow rate  Off-ramp flow rate

Density, Flow, Speed relation
For a segment $i$, the linear equations of the macroscopic traffic flow models are (Messner and Papageorgiou, 1990):

$$u_i(k+1) = u_i(k) + \frac{\Delta T}{\tau} \{ V[d_i(k)] - u_i(k) \} + \frac{\Delta T}{\Delta l} u_i(k)[u_{i-1}(k) - u_i(k)] - \frac{\nu}{\tau \Delta l} \frac{\Delta T}{d_i(k)} \frac{d_{i+1}(k) - d_i(k)}{d_i(k) + \kappa}$$

where:
- $u_i(k)$ is the static speed for segment $i$ at time $k$ with respect to the density $d_i(k)$.
- $V[d_i(k)] = u_f \cdot \exp[-\frac{1}{a} \left( \frac{d_i(k)}{d_c} \right)^a]$ is the speed impact function.

Note: $\tau$, $\kappa$, $\nu$ are the control parameters which need to be calibrated with field data.
VSL control Strategy

- The time interval for the traffic flow model is $\Delta t$ (i.e. 10s);
- The time interval for detector data update is $\Delta T$ (i.e. 1 min)
- The time interval for updating the VSL speed display is $T$ (i.e. 5 min)
A reactive Optimization model

- For a reactive model, the detected data (such as flow rate, speed) from previous time period will be used for control optimization, to be implemented in the next period.

- One major assumption of the reactive model is that traffic conditions would not change significantly in the next period.
The base proactive model

To implement the macroscopic traffic flow model to a VSL control system, one shall set the following additional constraints:

- For each subsection \( i \), the mean speed shall not exceed the displayed speed limit:

\[
\begin{align*}
\text{segment } i \text{ without VSL control:} & \quad u_j \leq u_i(k) \leq u_f, \\
\text{segment } i \text{ with VSL control:} & \quad u_j \leq u_i(k) \leq u_f v_i(k),
\end{align*}
\]

- Speed limit constraints

- Variable Speed Limit Ratio

- Density Constraints

- Constraint for the difference between two successive displayed VSLs
Base Control Model

Objective Functions:

1. Minimization of total travel time over the controlled segments:
\[
\min \sum_{k} \sum_{i} n_i d_i(k) \Delta T
\]

2. Minimization of speed variance along the target freeway:
\[
\min \sum_{k} \sum_{i} (u_i(k) - u_{ave})^2
\]
\[
u_{ave} = \frac{\sum_{k} \sum_{i} u_i(k)}{(T^c / \Delta T) \cdot N}
\]
The base proactive model

Disadvantages:

1. The prediction model may lose its accuracy over time.
2. The detector update interval $\Delta T < T$, and the prediction model cannot fully utilize the detector data.
A KF based proactive model

- To address these two issues, one can use KF to help correct the prediction over every $\Delta T$ interval.
Traffic measurements

- Consider a traffic detector installed at the boundary of two adjacent segments $i$ and $i+1$, for flow measurements from detector:

- Measurements of flow rate
  \[ m^q_i(k) = q_i(k) + \varepsilon^q_i(k), \]

- Measurements of on-ramp flow rate
  \[ m^r_i(k) = r_i(k) + \varepsilon^r_i(k), \]

- Measurements of off-ramp flow rate
  \[ m^s_i(k) = s_i(k) + \varepsilon^s_i(k), \]

- Measurements of average speed
  \[ m^u_i(k) = u_i(k) + \varepsilon^u_i(k), \]
Kalman Filter for Traffic State Estimation

Let $y_k$ denote the traffic state, including speed, density and flow rate, then the traffic flow estimation model could be represented as:

$$y_k = A y_{k-1} + B u_k + w_{k-1}$$

The detector measurements is:

$$z_k = H y_k + v_k$$

Also assume that:

$$w \sim N(0,Q)$$
$$v \sim N(0,R)$$
Kalman Filter for Traffic State Estimation

We define two variables:

- $\hat{y}_k^-$: Priori state estimate at step $k$ (before correction)
- $\hat{y}_k$: Posteriori state estimate at step $k$ (after correction)

And then a priori and a posteriori estimate errors could be defined as:

- $e_k^- = y_k - \hat{y}_k^-$
- $e_k = y_k - \hat{y}_k$

For the future computation, a priori estimate and a posteriori estimate error covariance are then

- $P_k^- = E[e_k^- e_k^{\top}]$
- $P_k = E[e_k e_k^{\top}]$
Kalman Filter for Traffic State Estimation

In deriving the equations for the Kalman filter, the following equation is given:

\[ \hat{y}_k = \hat{y}_k^- + K(z_k - H \hat{y}_k^-) \]

One form of the resulting \( K \) that minimize the posteriori estimate error is given by:

\[ K = P_k^- H^T (H P_k^- H^T + R)^{-1} \]
Kalman Filter for Traffic State Estimation

Prediction (Time Update)

1. Project the state ahead
   \[ \hat{y}^- (k) = f(\hat{y}(k-1), u(k-1), 0) \] (priori estimate state)

2. Project the error covariance ahead
   \[ P^- (k) = A P(k-1) A^T + W Q W^T \] (priori estimate error covariance)

Correction (Measurement Update)

1. Compute the Kalman Gain
   \[ K = P^- (k) H^T (H P^- (k) H^T + V R V^T)^{-1} \] (Blending factor)

2. Update estimate with measurement \( z_k \)
   \[ \hat{y}(k) = \hat{y}^- (k) + K (z(k) - h(\hat{y}^- (k), 0)) \] (posterior estimate state)

3. Update Error Covariance
   \[ P(k) = (I - K H) P^- (k) \] (posterior estimate error covariance)
Optimization Model

- Every $\Delta T$ sees, the updated detector data will be used to correct the prediction and an optimization model is developed to determine the VSL speed for the following $T$ sees.

- However, due to the fact that $\Delta T < T$, not every optimized speed would be displayed on VSL (for operational concerns).

- One can use the concept of “Rolling Horizon” to determine the display speed.
Rolling Horizon

The new displayed speed will be determined on the basis of optimized speeds over 1~5 sequential steps.
The new displayed speed

1) Define a counter \( M \) to identify the moving direction of the speed limit, and then denote \( v_i \) as the displayed speed limit of the current horizon, where \( M \) is updated by the following expression:

\[
M = \begin{cases} 
M + 1, & \text{if } v(i) > v' \\
M, & \text{if } v(i) = v', \quad i=1,2,\ldots,n; \\
M - 1, & \text{if } v(i) < v'
\end{cases}
\]

2) The new displayed speed limit for the next horizon will be readjusted with the predetermined increment \( \Delta \), based on the value of \( M \):

\[
v_{i+1}^' = \begin{cases} 
v' + \Delta, & \text{if } M > 0 \\
v', & \text{if } M = 0 \\
v' - \Delta, & \text{if } M < 0
\end{cases}
\]
System Framework

Freeway Segmentation
Locate VSL and Detectors

Initialization
\( t=0, M=0 \)

Kalman Filter

Start

Detector Data

Traffic Flow Model

Counter M update

t = t+1

Optimization Model

Yes

t < Tc ?

Control Strategy

Display new speed limit

No

Stop VSL?

Yes

Stop

No
Proposed Model Evaluation

VISSIM Simulation Tool

Initialization

External Data

Time period k=1

K<T

Yes

Optimization Module

New VSL

Simulation Continue

Stop

No

Yes (complete the entire simulation time period)

Terminate Simulation

No

Yes

Simulation Continue

Stop

Yes (complete the entire simulation time period)
**Numerical Example**

**Location:** the segment MD-100 West from MD 713 to Coca-Cola Drive;

**Time:** 6:00-8:00 AM;

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Field Observations

Freeway Segmentation

Detector

Free Flow

Bottleneck

VSL2

VSL1

Congestion (0.3 Mile)  Subsegment (0.9 Mile)  Free-Flow Segment
Numerical Example

To compare the proposed VSL models with No-control scenario, this study has designed the following four scenarios:

- **BASIC-TTT**: the base proactive model with the objective of total travel time minimization;
- **KF-TTT**: the enhanced proactive model with the objective of total travel time minimization;
- **BASIC-SV**: the base proactive model with the objective of speed variance minimization; and
- **KF-SV**: the enhanced proactive model with the objective of speed variance minimization.
Numerical Examples
### Numerical Examples

<table>
<thead>
<tr>
<th>Scenario</th>
<th>6:00-8:00 Ave. # of Stops</th>
<th>7:00-8:00 Ave. # of Stops</th>
<th>6:00-8:00 Ave. Travel Time (s)</th>
<th>7:00-8:00 Ave. Travel Time (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>No-VSL</td>
<td>6.34</td>
<td>10.49</td>
<td>214.6</td>
<td>302.9</td>
</tr>
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<td>BASIC-TTT</td>
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<td>9.70</td>
<td>211.0</td>
<td>294.0</td>
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<tr>
<td>KF-TTT</td>
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<td>7.61</td>
<td>188.5</td>
<td>257.3</td>
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<tr>
<td>BASIC-SV</td>
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<td>9.30</td>
<td>209.7</td>
<td>290.3</td>
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<tr>
<td>KF-SV</td>
<td>4.02</td>
<td>6.33</td>
<td>183.0</td>
<td>241.9</td>
</tr>
</tbody>
</table>
Conclusions

- A proper VSL system can effectively reduce the number of stops and travel time over a recurrently congested freeway segment;

- The macroscopic traffic flow model cannot fully capture the occurrence of congestion caused by traffic weavings, as reflected by the travel time and number of stops produced by BASIC-TTT;

- The accuracy of the predicted traffic congestion can significantly affect the effectiveness of a VSL control system, as evidenced by the superior performance of two models with an embedded KF functions;
Conclusions

- The Kalman Filter is proved as a useful tool in the VSL system implementation as its on-line traffic prediction function can effectively capture the complex traffic dynamics;

- Speed variance minimization seems to be a better control objective, as the speed data is directly measurable from detectors and is the most sensitive variable to the VSL control;

- Speed variance minimization model is less sensitive to the prediction accuracy, as reflected in the performance evaluation results.