Integrated Optimal Control and Diversion Algorithm for Congested Traffic Corridors -I

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Literature Reviewed


Formulation of Dynamic Traffic State Evolution

• Traffic State Evolution for Surface Street
• Traffic State Evolution on Freeway and Ramp Links

On-line Estimation of Model Parameters

• Estimation and Prediction of Turning Proportions
• Estimation and Prediction of Diversion Compliance Rates

Optimal Control Model

• Objective Functions
• Constraints

Solution Algorithm

• Inner-loop Iteration
• SLP Algorithm
• Outer-loop Iteration

Numerical Example

• Experimental Design
• Simulation Results
Problem definition

- The development of an integrated & coordinated traffic control model is a challenging task.
- In general, the formulations for optimal corridor control turns out to be large-scale, nonlinear, and non-differential, difficult to be solved efficiently for on-line applications.
Methodology

- adopted a simplified piecewise linear traffic flow model to approximate the traffic flow dynamic, which can lead to a tractable solution.
- proposed a successive linear programming algorithm to efficiently solve the optimal control problem.
System description

The left-hand side figure is the network used for the model formulation, including freeway with on-ramps and off-ramps and surface street.

Key variables & notations:

\[ A(l) \] set of links connected to the upstream node of link \( l \)

\[ B(l) \] set of outgoing approaches from the downstream node of link \( l \)

\[ q_m^l(k) \] flow rate entering section \( m+1 \) from section \( m \) of link \( l \) during interval \( k \)
### Definitions of Key variables

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\overline{q}^l_m(k)$</td>
<td>average flow rate in section $m$ of link $l$ during interval $k$</td>
</tr>
<tr>
<td>$d^l_m(k)$</td>
<td>mean traffic density in section $m$ of link $l$ during interval $k$</td>
</tr>
<tr>
<td>$f^l_m$</td>
<td>flow-density function in section $m$ of link $l$</td>
</tr>
<tr>
<td>$X_l(k)$</td>
<td>average effective green time/cycle length (g/C) ratio for link $l$ at its downstream intersection during interval $k$</td>
</tr>
<tr>
<td>$s^l$</td>
<td>saturation flow rate for queue discharging at the downstream end of link $l$</td>
</tr>
<tr>
<td>$r^l_{il}(k)$</td>
<td>fraction of leaving flows at the downstream end of link $l$ turning to link $l$</td>
</tr>
<tr>
<td>$Q^l(k)$</td>
<td>average number of vehicles queuing on link $l$ during interval $k$</td>
</tr>
</tbody>
</table>
### System description (Cont.)

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_l(k)$</td>
<td>metering flow rate at the downstream end of on-ramp link l</td>
</tr>
<tr>
<td>$Z_l(k)$</td>
<td>diversion control flow rate at freeway off-ramp link l</td>
</tr>
<tr>
<td>$\lambda_l(k)$</td>
<td>compliance rate of diversion control, which is a decimal fraction of $Z_l(k)$</td>
</tr>
</tbody>
</table>
Traffic state evolution on surface streets

- **Flow conversion law for each subsection**

\[ d^l_m(k) = d^l_m(k-1) + \left[ q^l_{m+1}(k) - q^l_m(k) \right] \cdot \Delta t / L_m \]

- Mean density of segment m of link l during interval k
- Flow rate entering segment m of link l during interval k
- Flow rate exiting segment m of link l during interval k

\( \Delta t \): duration of a unit time interval

\( n_m \): # of Lanes

\( L_m \): segment length
Traffic state evolution on surface streets

- **Computation of flow rate** $q_m^l(k)$

  - **Case 1:** if $q_m^l(k)$ is the transition flow from segment $m$ to segment $m+1$ within link $l$

    $$q_m^l(k) = \alpha \overline{q}_m^l(k) + (1 - \alpha) \overline{q}_{m+1}^l(k) \quad 1 \leq m < N(l), \forall l$$

    - $\alpha$ : weighting parameter
    - $N(l)$ : number of segments in link $l$

Flow rate exiting segment $m$ of link $l$ during interval $k$

**Average flow rate of segment $m$ in link $l$ during interval $k$**

**Average flow rate of segment $m+1$ in link $l$ during interval $k$**
Traffic state evolution on surface streets

- **Estimate the average segment flows** $\bar{q}_m^l(k)$
- The segment average flow is determined by an equilibrium flow density relationship

$$\bar{q}_m^l(k) = f_m^l\left[d_m^l(k)\right] \quad 1 \leq m < N(l), \forall l$$

- $\bar{q}_m^l(k)$ represents the functional form of the flow-density model, here it takes an inverted, V-shaped, two segment linear function
- $f_m^l$ represents the functional form of the flow-density model, here it takes an inverted, V-shaped, two segment linear function
- Mean density of segment m of link l during interval k
- Average flow rate of segment m of link l during interval k

**Traffic model formulation**

**Introduction**

- Estimate the average segment flows
- The segment average flow is determined by an equilibrium flow density relationship

**Parameter estimation**

**Optimal control**

**Solution algorithm**

**Numerical example**

**Conclusion**
Traffic state evolution on surface streets

- Computation of flow rate $q^l_m(k)$ (Cont.)
  - Case 2: for $m=1$, then $q^l_{m-1}(k) = q^l_0(k)$, which is the flow entering upstream boundary of link $l$.

\[
q^l_0(k) = \sum_{i \in A(l)} q^i_N(k) \cdot r_{il}(k) \cdot \mathcal{A}(l) \quad \text{set of links connecting to upstream node of link $l$}
\]

- Flow entering the upstream boundary of link $l$ during interval $k$
- Flow discharging from link $l$'s upstream adjacent links
- Turning ratio from link $i$ to link $l$
Traffic state evolution on surface streets

- **Computation of flow rate** \( q^l_m(k) \) (Cont.)
  - Case 2: for \( m=1 \), then \( q^l_{m-1}(k) = q^l_0(k) \), which is the flow entering upstream boundary of link \( l \).
  - If link \( l \) is the entry link of the corridor network, the entry flow is given directly by the O-D information.
Traffic state evolution on surface streets

- **Computation of flow rate** $q_m^l(k)$ (Cont.)

  - **Case 3**: $m = N(l)$, $q_m^l(k)$ is the discharging flow at the downstream node of link $l$.

\[
q_m^l(k) = \min \left\{ X_{ll}(k)(A_l + P_k) / \Delta t, S_l \right\}
\]

The first term indicates that the queue is sufficiently long and the saturation flow rate has been reached.

The second term indicates that the queue is not sufficiently long and the upcoming flow adding to the queue needs to be considered.
Traffic state evolution on surface streets

- **Computation of flow rate** $q_m^l(k)$ (Cont.)
  - **Case 3:** $m = N(l)$, $q_m^l(k)$ is the discharging flow at the downstream node of link $l$.
  - Approximate the discharging flow $Q_l(k-1)$ with the average number of vehicles in section $N(l)$.
  - $P_l(k)$ is approximated with the section boundary flow $q_{N(l)-1}^l(k)$ with a fraction determined by the time lag for the traffic to go through the last segment.

\[ \Delta t : \text{duration of a unit time interval} \]
\[ q_{N(l)}^l(k) = \min \left\{ s_l \cdot X_l(k), \left[ Q_l^{m^l(k)} + P_l(k) \right] \right\} \]
\[ L_{N(l)}^l : \text{segment length} \]
\[ t_l : \text{travel time to pass segment } N(l) \text{ without delay} \]
\[ q_{N(l)-1}^l : \text{flow exiting segment } N(l)-1 \]
Traffic state evolution on surface streets

- **Computation of flow rate** $q_m^l(k)$ (Cont.)

- Case 3: $m = N(l)$, $q_m^l(k)$ is the discharging flow at the downstream node of link $l$.

- If link $l$ is the exiting link of the corridor network, $q_N^l(k)$ can be easily taken as the average section flow $\overline{q}_N^l(k)$.
Traffic state evolution on freeway and ramp links

- Using the similar concept, the flow interaction between freeway (ramp) sections can be modeled with the same logic applied to surface streets.

- However, the model for the on-ramp merge and is different from the one used for off-ramp exit.
Traffic state evolution on freeway and ramp links

- Flow transition at an on-ramp merging node

- Discharging flow at the downstream end of each on-ramp (similar to the surface street model)

\[
q^{I''}_{N(I'')} (k) = \min \left[ R^{I''}_{i''}(k), d^{I''}_{N(I'')} \cdot L^{I''}_{N(I'')} \cdot n^{I''}_{N(I'')} / \Delta t + (1 - t_i / \Delta t) q^{I''}_{N(I'')} - 1 (k) \right]
\]

- where \( R^{I''}_{i''}(k) \) is the ramp metering flow rate.
Traffic state evolution on freeway and ramp links

- Flow transition at an on-ramp merging node
- Mainline transition flow at on-ramp merging
- 1) flow entering link $l'$

$$q''_0(k) = \alpha \left[ \overline{q}^l_{N(l)}(k) + q''_{N(l'')} (k) \right] + (1-\alpha) \overline{q}_{l''}(k)$$

- flow entering link $l'$ during interval $k$
- average flow of segment $N(l)$ of link $l$ during interval $k$
- discharging flow of segment $N(l'')$ of ramp link $l''$ during interval $k$
- average flow of link $l'$ during interval $k$
Traffic state evolution on freeway and ramp links

- Flow transition at an on-ramp merging node (Cont.)
- Mainline transition flow at on-ramp merging

2) flow exiting link \( l \)

\[
q^l_{N(l)}(k) = \alpha \overline{q}^l_{N(l)}(k) + (1 - \alpha) \left[ q^0_0(k) - q^l_{N(l'')}(k) \right]
\]

- Flow exiting link \( l \) during interval \( k \)
- Average flow of segment \( N(l) \) of link \( l \) during interval \( k \)
- Flow entering link \( l' \) during interval \( k \)
- Discharging flow of segment \( N(l'') \) of ramp link \( l'' \) during interval \( k \)
Traffic state evolution on freeway and ramp links

- Flow transition at an off-ramp merging node

Actual flow leaving the off-ramp

\[ q_{0}^{l''}(k) = r_{l''}(k) \cdot q_{N(1)}^{l}(k) + \lambda_{l}(k)Z_{l''}(k) \]
Traffic state evolution on freeway and ramp links

- Flow transition at an off-ramp merging node (Cont.)
- Mainline transition flow at
- 1) flow exiting link /

\[ q_{N(l)}^i(k) = \alpha q_{N(l)}^i(k) + (1 - \alpha) \left[ q_{(l')}^i(k) + q_{(l'')}^i(k) \right] \]

- Flow exiting link / during interval k
- Average flow of segment \( N(l) \) of link / during interval k
- Average flow of link /" during interval k
- Average flow of ramp link /" during interval k
Traffic state evolution on freeway and ramp links

Flow transition at an off-ramp merging node (Cont.)

Mainline transition flow at off-ramp merging

2) flow entering link $l'$

\[
q_0^{l'}(k) = \alpha \left[ 1 - r_{ll}^{"'}(k) \right] q_N^{l}(k) + (1 - \alpha) \left[ 1 - r_{ll}^{"'}(k) \right] \left[ q_{(l)}(k) + q_{(l')}^{"'}(k) \right] - \lambda_l(k) \cdot Z_{ll}^{"'}(k)
\]
Traffic state evolution

- The entire corridor model for traffic flow evolution

\[
d(k) = F\left[d(k-1), X(k), R(k), Z(k), \lambda(k), r(k), E(k)\right]
\]

- where

\[
F \quad \text{functional form determined by the above equations}
\]
\[
d(k) \quad \text{density distribution vector}
\]
\[
X(k), R(k), Z(k) \quad \text{control parameter vectors denoting g/C ratio, ramp metering rate and diversion rate, respectively}
\]
\[
\lambda(k), r(k), E(k) \quad \text{system input variables, representing compliance rates, turning ratios and network entry flows}
\]
On-line estimation of model parameters

- The methods to estimate the current density, based on on-line traffic measurements, are available in literatures from Payne et al\[^1\].
- The demand is assumed to be given from external sensors.
- The parameters needs to be estimated are turning ratios and diversion compliance rates.

Estimation and prediction of turning ratios

- Generally, the estimation of turning ratios can be completed based on the least-square estimation and Kalman filter techniques.
- Prediction of turning ratios may be made through some time-series models, such as ARIMA.
- Here, because of the correlation between diversion flows and turning ratios, we need a feedback mechanism to construct the relationship between these two.
Based on the turning ratio obtained, the compliance rate could be calculated as the ratio between actually observed diverting flows and assumed diversion flows.

\[
\lambda_i(k) = \left[ q_0''(k) - r_{ll''}q_{N(l)}^{l'}(k) \right] / Z_i(k)
\]
The total travel time (TTT) is selected as the objective function, which is expressed as:

$$\min TTT = \sum_k \left\{ \sum_l \sum_{m=1}^{N(l)} \left[ d_m^l(k) \cdot L_m^l \cdot n_m^l \right] \right\} \cdot \Delta t$$

where $d_m^l(k) \cdot L_m^l \cdot n_m^l$ represents the average number of vehicles on section $m$ of link $l$ during time interval $k$.
Constraints

- The dynamic traffic state evolution models serve as the primary constraints.
- Besides, there are some physical constraints for the density and control variables, such as:
  \[ 0 \leq d^l_m(k) \leq d^\text{max}, \quad \forall m, l \quad (1) \]
  \[ 0 \leq \lambda_i(k) \cdot Z^l_i(k) \leq s^l_i - r_l^\text{off-ramp} q^l_N(k), \quad \forall \text{ off-ramp } l'' \quad (2) \]
- (2) implies that the diversion flow plus the original off-ramp flow cannot be larger than the saturation flow of the off-ramp
  \[ R^l,\text{min} \leq R_i^l(k) \leq R^l,\text{max}, \quad \forall \text{ on-ramp } l \quad (3) \]
- where \( R^\text{min} \) and \( R^\text{max} \) is the minimum and maximum metering rate
Constraints (Cont.)

\[ X_{l,\min} \leq X_l(k) \leq X_{l,\max}, \quad \forall l \quad (4) \]

- where \( X_{\min} \) and \( X_{\max} \) is the minimum and maximum allowed \( g/C \) ratios.

\[ X_l(k) + \sum_{l' \in B(l)} X_{l'}(k) = 1 + \sigma_l - \phi_l, \quad \forall l \quad (5) \]

- where:
  - \( B(l) \) set of approaches to the downstream intersection of link \( l \)
  - \( \sigma \) representing the green-phase overlapping factor among the approaches in signal phasing
  - \( \phi \) decimal fraction of lost time due to start-up delays and signal changes
Flow-density relationship

- The optimization model would be piecewise linear if the flow-density relationship is piecewise linear.
- A inverted, V-shape, two segment linear function is adopted to approximate the flow-density relationship.
The two-segment linear flow-density function is expressed as

\[ q = \begin{cases} 
(q^*/d^*)d & \text{if } 0 \leq d \leq d^* \\
q^*(d - d_{\text{max}})/(d^* - d_{\text{max}}) & \text{if } d^* \leq d \leq d_{\text{max}} 
\end{cases} \]

\[ q = \min \left[ \left( q^*/d^* \right) d, q^* \frac{(d - d_{\text{max}})}{(d^* - d_{\text{max}})} \right] \]
Flow-density relationship (Cont.)

- The two-segment linear flow-density function is expressed as
  \[ q = \min \left( \frac{q^*}{d^*} d, \frac{q^*}{d^*} (d - d_{\text{max}})/\left(\frac{d^*}{d_{\text{max}}} - 1\right) \right) \]

- Introduce a set of binary variables to define which part of the flow-density function should be used
  \[ \delta_i = \begin{cases} 
  1, & \text{if in free-flow regime} \\
  0, & \text{if in congested regime} 
\end{cases} \]
Contents of the solution algorithm

- The complete solution algorithm includes an outer-loop and an inner-loop.
- In the inner-loop, a successive linear programming algorithm is proposed to find the optimal solution with fixed turning proportions.
- In the outer-loop, the turning proportion may be revised due to the new control solution for the diversion flows.
Flowchart of the solution algorithm

FIGURE 5 Solution algorithm.
Successive linear programming algorithm

- Step 0: Given an initial feasible solution $Y_0$, determine the respective values of $\delta_i$ for all the segment at all time interval. Specify a precision threshold $\varepsilon$. Set $n=0$.

- Step 1: Under the current value of $\delta_i^{n-1}$, solve the optimization problem and obtain the optimal solution $Y^n$.

- Step 2: Check whether the new value of objective function is much smaller than the previous value, $D^T Y^n - D^T Y^{n-1} < -\varepsilon$. If not, stop this procedure and $Y^n$ is the optimal solution.
Successive linear programming algorithm (Cont.)

- Step 3: Determine the set $E^n$ whose flow-density relationship changes from one regime to another.
- Step 4: Let $\delta_i^n = 1 - \delta_i^{n-1}$ for all $i \in E^n$, set $n = n+1$ and return to step 1.
Features of the SLP Algorithm

- Convenient for implementation (only solving LPs)
- LP solution at each step improves the result monotonically
- Convergent within a limited number of iterations
- The Maximum possible number of LP problems need to be solved has an upper bound.
Outer-loop iteration

- The outer-loop iteration revises the turning proportion based on the diversion flow rate obtained from the inner-loop solution.
- If the diversion flow rate is not significantly different from the one obtained from last outer-loop iteration, the algorithm should be terminated and the optimal solution is obtained.
Outer-loop iteration (Cont.)

- Assume a link has three turning movements, left-turn, through and right, denoted as \( r_1, r_2, r_3 \).
- The link flow prior to the diversion is \( q \) and the additional diverted flow is \( \Delta q \).
- The turning proportion is revised as:
  
  \[
  r_1 = \frac{r_1 q}{(q + \Delta q)} \\
  r_2 = \frac{r_2 q + \Delta q}{(q + \Delta q)} \\
  r_3 = \frac{r_3 q}{(q + \Delta q)}
  \]
A sample corridor network is shown in the following figure.

- incident occurs on section 8
- traffic control includes:
  - ramp metering at on-ramps 22 & 27
  - signal timing for intersection 15 & off-ramp 25
  - flows diversion for section 4
Experimental design

- Four traffic conditions have been selected to be tested
  - Case 1: 75% saturation volume and incident level 25% capacity reduction
  - Case 2: 75% saturation volume and incident level 45% capacity reduction
  - Case 3: 90% saturation volume and incident level 25% capacity reduction
  - Case 4: 90% saturation volume and incident level 45% capacity reduction
Experimental design

- **Three control strategies:**
  - **Control A:** Baseline control, g/C ratio for approach 15 and 25 fixed at 0.6 and 0.4. No ramp metering and diversion control.
  - **Control B:** Long Island two layer control logic[2].
  - **Control C:** Proposed model.

## Simulation results

- TTT (veh-min)

<table>
<thead>
<tr>
<th>Control Strategy</th>
<th>Case 1</th>
<th>Case 2</th>
<th>Case 3</th>
<th>Case 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Strategy A</td>
<td>3445</td>
<td>3788</td>
<td>4375</td>
<td>4826</td>
</tr>
<tr>
<td>Strategy B</td>
<td>3416</td>
<td>3654</td>
<td>4348</td>
<td>4451</td>
</tr>
</tbody>
</table>
The two control strategies both work with respect to average speed, but the proposed model outperforms the other one under all the cases.

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<th>Case 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Strategy A</td>
<td>55.3</td>
<td>50.1</td>
<td>47.5</td>
<td>43.0</td>
</tr>
<tr>
<td>Strategy B</td>
<td>55.8</td>
<td>52.9</td>
<td>49.4</td>
<td>49.5</td>
</tr>
</tbody>
</table>
Simulation results

- **TTD (veh-mile)**

<table>
<thead>
<tr>
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<th>Case 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Strategy A</td>
<td>3175</td>
<td>3163</td>
<td>3463</td>
<td>3458</td>
</tr>
<tr>
<td>Strategy B</td>
<td>3711</td>
<td>3222</td>
<td>3580</td>
<td>3672</td>
</tr>
</tbody>
</table>
Conclusion

- Based on the simplified flow-density relationship, a SLP algorithm is proposed to efficiently solve the optimization problem.
- The formulations of dynamic traffic state evolution consider the flow transition between adjacent sections.
- The turning ratio is no longer assumed to be constant and is updated based on the diversion flow rate.
- The result shows that the proposed model is effective under non-recurrent congestion.
Thank you & Questions?