Integrated Variable Speed Limit and Ramp Metering Control for Managing Recurrent Freeway Congestion
----Local Bottleneck Control

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Outline

- Control Flowchart
- Notations
- Determine the System Activation Time
- Determine the activation time of VSL Control
- Compute the Targeted Flow Rate Reduction
- Decide the Number of VSLs to be Activated
- Distribute Flow Rate Reduction to the On-ramp and Mainline
- Refinement-stage Optimization
Control Flowchart

Initialization

Reach the $k_r$th RM control interval

Yes

Predicted unstable or congested states?

Yes

Optimize Ramp Metering Rate based on Local Search Algorithm

No

No

Select the Maximize RM

$k_r = k_r + 1$

$k = k + 1$

$k_v = k_v + 1$

System Activation?

Yes

Yes

Select the Maximum RM & VSL

No

No

Determine the Starting Points of VSL Control

Determine the Number of VSLs to be Activated

Refinement-stage Optimization

Distribute Flow Rate Reduction to On-ramps and Mainline

No

Compute the Targeted Flow Rate Reduction

$k_v = k_v + 1$

$k = k + 1$

$k_r = k_r + 1$

$k_v = k_v + 1$

$k = k + 1$
## Notations

<table>
<thead>
<tr>
<th>Variables</th>
<th>Explanations</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta t$</td>
<td>Time step for updating traffic conditions (10 s)</td>
</tr>
<tr>
<td>$n_1$</td>
<td>Number of $\Delta t$ in one VSL control interval ($n_1=30$)</td>
</tr>
<tr>
<td>$n_2$</td>
<td>Number of $\Delta t$ in one ramp metering control interval ($n_2=6$)</td>
</tr>
<tr>
<td>$n_p$</td>
<td>Number of $\Delta t$ in one prediction horizon ($n_p=30$)</td>
</tr>
<tr>
<td>$t_v = n_1\Delta t$</td>
<td>Length of one VSL control interval (5 min)</td>
</tr>
<tr>
<td>$t_r = n_2\Delta t$</td>
<td>Length of one ramp metering control interval (1 min)</td>
</tr>
<tr>
<td>$t_p = n_p\Delta t$</td>
<td>Length of one prediction interval (5 min)</td>
</tr>
</tbody>
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<table>
<thead>
<tr>
<th>Variables</th>
<th>Explanations</th>
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<tbody>
<tr>
<td>$k$</td>
<td>Time step index of traffic flow models, where $k$ th interval $= k \Delta t$</td>
</tr>
<tr>
<td>$k_v$</td>
<td>Time step index of VSL control, where $k_v$ th interval $= k_v t_v$</td>
</tr>
<tr>
<td>$k_r$</td>
<td>Time step index of ramp metering control, where $k_r$ th interval $= k_r t_r$</td>
</tr>
<tr>
<td>$i$</td>
<td>Index of freeway segments</td>
</tr>
<tr>
<td>$b$</td>
<td>Index of the bottleneck</td>
</tr>
<tr>
<td>$d$</td>
<td>Index of the most downstream segment in the VSL control area</td>
</tr>
<tr>
<td>$N$</td>
<td>Number of segments in the network</td>
</tr>
<tr>
<td>$M$</td>
<td>A very big number used for penalty purpose</td>
</tr>
</tbody>
</table>
### Notations (Cont.)

<table>
<thead>
<tr>
<th>Variables</th>
<th>Explanations</th>
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</thead>
<tbody>
<tr>
<td>$L_i$</td>
<td>Length of segment $i$ (km)</td>
</tr>
<tr>
<td>$\lambda_i$</td>
<td>Number of lanes on segment $i$</td>
</tr>
<tr>
<td>$\rho_i(k)$</td>
<td>Density of segment $i$ at time step $k$ (veh/km/ln)</td>
</tr>
<tr>
<td>$v_i(k)$</td>
<td>Speed of segment $i$ at time step $k$ (km/h)</td>
</tr>
<tr>
<td>$q_i(k)$</td>
<td>Flow rate out of segment $i$ at time step $k$ (veh/h)</td>
</tr>
<tr>
<td>$v_{f,i}(k)$</td>
<td>Posted speed limit of segment $i$ at time step $k$ (km/h)</td>
</tr>
<tr>
<td>$\rho_{c,i}$</td>
<td>Critical density of segment $i$ (veh/km/ln)</td>
</tr>
<tr>
<td>$a_i, \tau_i, u_i, \kappa_i$</td>
<td>Parameters in traffic flow models for segment $i$</td>
</tr>
</tbody>
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## Notations (Cont.)

<table>
<thead>
<tr>
<th>Variables</th>
<th>Explanations</th>
</tr>
</thead>
<tbody>
<tr>
<td>$d(k)$</td>
<td>On-ramp demand at time step $k$ (veh/h)</td>
</tr>
<tr>
<td>$r(k)$</td>
<td>On-ramp flow rate at time step $k$ (veh/h)</td>
</tr>
<tr>
<td>$R(k)$</td>
<td>Ramp metering rate at time step $k$ (veh/h)</td>
</tr>
<tr>
<td>$w(k)$</td>
<td>On-ramp queue length at time step $k$ (veh)</td>
</tr>
<tr>
<td>$C_r$</td>
<td>On-ramp capacity (veh/h)</td>
</tr>
<tr>
<td>$w_{\text{max}}$</td>
<td>Maximum allowed queue length at the on-ramp</td>
</tr>
<tr>
<td>$R_q(k)$</td>
<td>Ramp metering rate resulting in the maximum allowed queue at time step $k$ (veh/h)</td>
</tr>
<tr>
<td>Variables</td>
<td>Explanations</td>
</tr>
<tr>
<td>---------------</td>
<td>--------------</td>
</tr>
<tr>
<td>$\Delta Q_b(k_v)$</td>
<td>Targeted reduction on the flow rate from the control area at the $k_v$ th VSL control interval (veh/h)</td>
</tr>
<tr>
<td>$Q_b(k_v)$</td>
<td>Estimated average outflow rates from the control area at the $k_v$ th VSL control interval (veh/h)</td>
</tr>
<tr>
<td>$C_b$</td>
<td>Bottleneck capacity (veh/h)</td>
</tr>
<tr>
<td>$\rho_b(k)$</td>
<td>Bottleneck density at time step $k$ (veh/km/ln)</td>
</tr>
<tr>
<td>$\rho_{c,b}$</td>
<td>Bottleneck critical density (veh/km/ln)</td>
</tr>
<tr>
<td>$K$</td>
<td>Gain factor used in the VSL control</td>
</tr>
<tr>
<td>$\Delta v$</td>
<td>The maximum speed reduction for one VSL control interval (km/h)</td>
</tr>
</tbody>
</table>
### Notations (Cont.)

<table>
<thead>
<tr>
<th>Variables</th>
<th>Explanations</th>
</tr>
</thead>
<tbody>
<tr>
<td>$v_l$</td>
<td>The lowest speed allowed to be displayed (km/h)</td>
</tr>
<tr>
<td>$S^j$</td>
<td>Segment set controlled by VSL $j$</td>
</tr>
<tr>
<td>$N^j$</td>
<td>Number of segments in set $S^j$</td>
</tr>
<tr>
<td>$Q^j(k_v)$</td>
<td>Estimated average outflow rates from the control area at the $k_v$ th VSL control interval if the $j$ th VSL is activated (veh/h)</td>
</tr>
<tr>
<td>$\Delta Q^j(k_v)$</td>
<td>Reduction on the flow rate from the control area at the $k_v$ th VSL control interval if the $j$ th VSL is activated (veh/h)</td>
</tr>
<tr>
<td>$\bar{\rho}^j(k)$</td>
<td>The resulting average density over the segments controlled by VSL $j$ at time step $k$ (veh/km/ln)</td>
</tr>
</tbody>
</table>
## Notations (Cont.)

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<thead>
<tr>
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<th>Explanations</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\overline{\rho}^j_c$</td>
<td>Average critical density over the segments controlled by VSL $j$ (veh/km/ln)</td>
</tr>
<tr>
<td>$J_{\text{max}}$</td>
<td>Maximum number of VSLs to be activated</td>
</tr>
<tr>
<td>$S$</td>
<td>Solution set with different active VSLs</td>
</tr>
<tr>
<td>$\Delta Q^j_R(k_v)$</td>
<td>Flow rate reduction distributed to on-ramps when the $j$ th VSL is activated at the $k_v$ th VSL control interval (veh/h)</td>
</tr>
<tr>
<td>$R^j(k)$</td>
<td>Ramp metering rate at time step $k$ when the $j$ th VSL is activated (veh/h)</td>
</tr>
<tr>
<td>$\delta(k)$</td>
<td>Penalty added to the solution at time step $k$</td>
</tr>
</tbody>
</table>
Determine the System Activation Time

- If the network is predicted to encounter congestion within the prediction horizon, it is the time to activate the control system.
Determine the Starting Point of the VSL Control

- The mainline control should start from the upstream of the bottleneck, not currently covered by congestion.
Targeted Flow Rate Reduction

- The targeted flow rate reduction for the $k_v$ th VSL control interval can be determined by two components:
  1) Difference between the estimated flow rate and the bottleneck capacity
  2) A modification module
Targeted Flow Rate Reduction (Cont.)

\[ \Delta Q_b(k_v) = Q_b(k_v) - C_b + K\{\rho_b(n_1(k_v-1)) - \rho_{c,b}\} \]

- Module 1 equals to the difference between the estimated flow rate from the control area and the bottleneck capacity.
The prediction errors may cause the resulting bottleneck density to deviate from the critical density.

Module 2 is to maintain the critical density based on the feedback mechanism.
Targeted Flow Rate Reduction (Cont.)

- If $\Delta Q_b(k_v) > 0$
  - The speed of VSLs shall be reduced at the next VSL control interval.

- If $\Delta Q_b(k_v) < 0$
  - The speed of VSLs shall be increased at the next VSL control interval.
Targeted Flow Rate Reduction (Cont.)

- Due to the compliance rate issue, the maximum speed reduction for one control interval is $\Delta v$ (for example, 10km/h).

- The lowest speed allowed to be displayed is $v_l$.

- For illustration, three VSLs are assumed to be present in the control area.
Determine the number of VSLs to be activated

- Based on the targeted flow rate reduction and the operational constraints to determine the maximum number of such devices to be activated.
If the speeds of VSL 1, VSL 2, VSL 3 at the \( k_v \) th control interval keep the same as the one at the \( k_v - 1 \) th VSL control interval, then set:

\[
v_{f,i}(k) = v_{f,i}(n_1(k_v - 1))
\]

for \( i \in S^j, j = 1:3, k = n_1(k_v - 1) + 1: n_1 k_v \)
The traffic dynamics can be calculated by [1]:

\[
\rho_i(k+1) = \rho_i(k) + \frac{\Delta t}{\lambda_i L_i} [q_{i+1}(k) - q_i(k)]
\]

\[
V[\rho_i(k)] = v_{f,i}(k) \exp \left[ -\frac{1}{a_i} \left( \frac{\rho_i(k)}{\rho_{cr,i}} \right)^a \right]
\]

\[
q_i(k) = \lambda_i \rho_i(k) v_i(k)
\]

\[
v_i(k+1) = v_i(k) + \frac{\Delta t}{\tau_i} [V(\rho_i(k)) - v_i(k)] + \frac{\Delta t}{L_i} v_i(k) [v_{i+1}(k) - v_i(k)] - \frac{u_i \Delta t}{\tau_i L_i} \frac{\rho_i(k) - \rho_{i-1}(k)}{\rho_i(k) + \kappa_i}
\]

Estimated flow rate if no VSLs are activated

- The average outflow rate (per hour) from the control area at the $k_v$th control interval can be computed and denoted as $Q_b(k_v)$

$$Q_b(k_v) = \frac{1}{n_1 k = n_1(k_v - 1) + 1} \sum_{n_1 k} \left[ q_d(k) + r(k) \right], d \in S^1$$
Flow Rate Reduction for VSL 1 Only

- If the speed of VSL-1 is reduced by $\Delta v$

$$v_{f,1}(k) = \max \left\{ v_{f,1}(n_1(k_v - 1)) - \Delta v, v_l \right\}, k = n_1(k_v - 1) + 1: n_1k_v$$

$$v_{f,2}(k) = \max \left\{ v_{f,2}(n_1(k_v - 1)) - \Delta v, v_l \right\}, k = n_1(k_v - 1) + 1: n_1k_v$$

- To compress the notations, the above equations are written as follows:

$$v_{f,i}(k) = \max \left\{ v_{f,i}(n_1(k_v - 1)) - \Delta v, v_l \right\}, k = n_1(k_v - 1) + 1: n_1k_v, i \in S^l$$
The average outflow rate (per hour) from the control area at the $k_v$ th VSL control interval can be computed as before and denoted as $Q^1(k_v)$.

Compared with the no control case, the flow rate reduction should be:

$$\Delta Q^1(k_v) = Q_b(k_v) - Q^1(k_v)$$
Flow Rate Reduction for VSL 1 Only

- Under either of these two conditions, the speed of VSL-2 may also be reduced:
  1) $\Delta Q^1(k_v) < \Delta Q_b(k_v)$, the flow rate reduction is not sufficient to prevent congestion
  2) $\bar{\rho}^1(k) = \sum_{i \in S^1} \rho_i(k)L_i \left/ \sum_{i \in S^1} L_i \right. > \bar{\rho}_c^1, k = n_1(k_v - 1) + 1 : n_1k_v$

- In a general scenario, the average critical density can be obtained as follows:
  $$\bar{\rho}_c^1 = \sum_{i \in S^1} \rho_{c,i}L_i \left/ \sum_{i \in S^1} L_i \right.$$
If the speeds of VSL-1 & VSL-2 are both reduced by $\Delta v$

$$v_{f,i}(k) = \max\{v_{f,i}(n_1(k_v - 1)) - \Delta v, v_i\}, \quad k = n_1(k_v - 1) + 1; \quad n_1, k_v, k \in Z, i \in S^1 \cup S^2$$

The average outflow rate from the control area at the $k_v$ th VSL control interval can be calculated and denoted as $Q^{1,2}(k_v)$

Flow rate reduction is:

$$\Delta Q^{1,2}(k_v) = Q_b(k_v) - Q^{1,2}(k_v)$$
Flow Rate Reduction for VSL 1 & VSL 2

Under either of these two conditions, the speed of VSL-3 may also be reduced:

1) $\Delta Q_{1,2}(k_v) < \Delta Q_b(k_v)$, the flow rate reduction is not sufficient to prevent congestion

2) $\bar{\rho}^{1,2}(k) = \frac{\sum_{i \in S^1 \cup S^2} \rho_i(k)L_i}{\sum_{i \in S^1 \cup S^2} L_i} > \bar{\rho}_{c}^{1,2}, k = n_i(k_v - 1) + 1; n_i k_v$

the average density within the area controlled by VSL-1 & 2 is larger than the critical density
Flow Rate Reduction for VSL 1, VSL 2, & VSL 3

Following the same procedure:

- Speeds for VSL 1 & VSL 2 & VSL 3 are all reduced:

\[ v_{f,i}(k) = \max \left\{ v_{f,i}(n_1(k_v - 1)) - \Delta v_v, v_i \right\} \]

where, \( k = n_1(k_v - 1) + 1: n_1 k_v, k \in \mathbb{Z}, i \in S^1 \cup S^2 \cup S^3 \)

- Resulting flow rate reduction can be computed as:

\[ \Delta Q^{1,2,3}(k_v) = Q_b(k_v) - Q^{1,2,3}(k_v) \]
Flow Rate Reduction for VSL 1, VSL 2, & VSL 3

- Average density within the control area is:

$$\overline{\rho}_{1,2,3}(k) = \frac{\sum_{i \in S^1 \cup S^2 \cup S^3} \rho_i(k)L_i}{\sum_{i \in S^1 \cup S^2 \cup S^3} L_i}, \ k = n_1(k_v, -1) + 1: n_1k_v$$

- The above procedure can be generalized to $J$ VSLs.
Flowchart to Determine the maximum number of VSLs without RM

1. **Initialization**
   - Set $J_{\text{max}} = 1$
   - $k = k + 1$

2. **Reach the $k_v$ th VSL control interval**
   - Yes: $p^{1,2}(k) \leq \overline{p}^{1,2}_c$
     - Yes: $\Delta Q^{1,2}(k_v) > \Delta Q_{\text{ref}}(k_v)$
       - No: $k_v = k_v + 1$
     - No: $k_v = k_v + 1$
   - No: $k_v = k_v + 1$

3. **Evaluate the effect of reducing the speed of VSL-1**
   - Yes: $\Delta Q^1(k_v) > \Delta Q_{\text{ref}}(k_v)$
   - Yes: Set $J_{\text{max}} = 2$
   - No: $k_v = k_v + 1$

4. **Evaluate the effect of reducing the speeds of VSL-1 & 2**
   - Yes: $\Delta Q^{1,2}(k_v) > \Delta Q_{\text{ref}}(k_v)$
   - Yes: Set $J_{\text{max}} = 3$
   - No: $k_v = k_v + 1$

5. **Determine the starting point of the control area**
   - No: $\Delta Q^{1}(k_v) > \Delta Q_{\text{ref}}(k_v)$
   - No: $k_v = k_v + 1$

6. **Evaluate the effect of reducing the speeds of VSL-1, 2, & 3**
   - Yes: $\Delta Q^{1,2,3}(k_v) > \Delta Q_{\text{ref}}(k_v)$
   - No: $k_v = k_v + 1$

7. **Set $J_{\text{max}} = 3$**

Note: $\rho = \frac{c}{v}$, where $c$ is capacity and $v$ is speed. 

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**Equations**

- $p^{1,2}(k) \leq \overline{p}^{1,2}_c$
- $\Delta Q^{1,2}(k_v) > \Delta Q_{\text{ref}}(k_v)$
- $\Delta Q^{1}(k_v) > \Delta Q_{\text{ref}}(k_v)$
- $\Delta Q^{1,2,3}(k_v) > \Delta Q_{\text{ref}}(k_v)$
The on-ramp queue, \( w(k) \) needs to be less than the maximum allowed queue length, \( w_{\text{max}} \).

Using the above equations, the corresponding maximum ramp metering rate at step \( k \), denoted as \( R_q(k) \), can be obtained as follows:

\[
R_q(k) = d(k) - \frac{w_{\text{max}} - w(k)}{\Delta t}
\]

VSL & RM Coordination Algorithm

- Step 1: Set \( j = J_{\text{max}} \), where \( J_{\text{max}} \) is the index of the VSLs to be activated in the \( k_v \) th control interval at the most upstream.

- Step 2: If \( \bar{\rho}^{1,...,j}(k) \leq \bar{\rho}_c^{1,...,j} \), add the corresponding control plan to the solution set (Let \( S = S \cup j \)), where speeds are reduced for VSL-1 to VSL-\( j \), the flow rate reduction is denoted as \( \Delta Q^{1,...,j}(k_v) \).

\[ k = n_1(k_v - 1) + 1 : n_1 k_v \]
VSL & RM Coordination Algorithm

- Step 3: Decrease \( j \) by one; if \( j > 0 \), go back to Step 2; else, go to Step 4.

- Step 4: For each solution added in Step 2, assign the remaining flow rate to the on-ramp.

\[
\Delta Q^1_{R}^{j..j}(k_v) = \Delta Q_b(k_v) - \Delta Q^{1..j}(k_v), \quad j \in S
\]
Step 5: Make sure that the resulting RM rate is no less than the one leading to the queue spillback.

\[ R^{1 \ldots j}(k) = \max \left\{ R(n_i(k_v - 1)) - \Delta Q^k_{R^{1 \ldots j}}(k_v), R_q(k) \right\} \]

where, \( k = n_i(k_v - 1) + 1; n_i k_v, j \in S \)
Select the Combination with the Best Objective Function Value

- **Step 1**: From the previous step, obtain different combinations of RM rate and displayed speeds of VSLs.

- **Step 2**: Determine the current objective function, which is to minimize TTT or maximize TTD.

- **Step 3**: Based on the selected objective function, compute the objective function for different combinations.

- **Step 4**: Select the combination with the best objective function value to enter the refinement stage to achieve better results.
Refinement-Stage Optimization

Initialization

Reach the $k_r$th RM control interval

Predicted unstable or congested states?

Yes

Yes

Select the maximum RM & VSL

Select the maximum RM

Unstable and congested states be avoided?

No

No

Adjust RM & VSL to minimize TTT

Adjust RM to minimize TTT

Adjust RM to maximize TTD and prevent unstable and congested states

Adjust RM & VSL to maximize TTD and prevent unstable and congested states

$k_r = k_r + 1$

$k = k + 1$

No

No

Predicted unstable or congested states?

No

Yes

Adjust RM & VSL to maximize TTD and prevent unstable and congested states

Adjust RM to maximize TTD and prevent unstable and congested states

$k = k + 1$

$k = k + 1$

$k_v = k_v + 1$
The objective function is first set to maximize the total travel distance, as long as the breakdown can be prevented through the control (Scenario 1).

If the breakdown becomes inevitable due to either the temporary demand surge or the prediction errors, the objective function is switched to minimize the total travel time (Scenario 2).
Scenario 1:

- Unstable and congested states can be prevented during the prediction horizon with the proper control strategies.

- Control Purposes:
  - delay the onset of traffic breakdown
  - Expedite the recovery after congestion

- The objective function is set to:
  - maximize the throughput (total travel distance)
  - And also prevent the traffic breakdown for every segment within the network
Refinement-stage Optimization Algorithm (Cont.)

※ Scenario 1

- Consider both equity and efficiency
- When the traffic breakdown can be prevented, try to reduce the waiting times for vehicles at the on-ramp
  - Although more vehicles may be present in the mainline, travel time will not be significantly affected.
  - Minimize the possibility of activating the ramp queue override mechanism, thus also improve mobility.
Objective function of Scenario 1:

\[
\text{Min } -TTD = -\Delta t \sum_{k=k_0+1}^{k_0+n_p} \sum_{i=1}^{N} \left[ \rho_i(k) v_i(k) \lambda_i L_i \right] + \delta(k_0)
\]


Operational constraints for RM and VSLs

\[
\delta(k_0) = \begin{cases} 
0 & \text{if all } \rho_i(k) < \rho_{c,i}, w(k) < w_{\max}, k = k_0 + 1 \cdot k_0 + n_p, i = 1: N \\
\delta(k_0) + M & \text{if any } \rho_i(k) \geq \rho_{c,i}, w(k) \geq w_{\max}, k = k_0 + 1 \cdot k_0 + n_p, i = 1: N 
\end{cases}
\]

Decision variables: RM rate & displayed speeds of VSLs
Refinement-stage Optimization Algorithm (Cont.)

Core logic:

- Starting from $k_0$, which is the current time step, calculate the total travel distance for the next $n_p$ steps.
- For any $k$ within the $n_p$ steps, if breakdown appears at any segment or ramp queue reaches the maximum value, a large penalty $M$ is added to the penalty term $\delta(k_0)$.
- If no penalties are added for several solution sets, the one with the maximum throughput is chosen.
Scenario 2:

- If unstable and congested states are inevitable due to either the temporary demand surge or the prediction inaccuracy, the objective function is then switched to minimize delay (total travel time) for traffic on the mainline and at the on-ramp.

\[
\text{Min} \quad TTT = \Delta t \sum_{k=k_0+1}^{k_0+n_p} \sum_{i=1}^{N} \left[ \rho_i(k) \lambda_i L_i + w(k) \right]
\]


Operational constraints for RM and VSLs
Thank you.
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