11 Antenna

11.1 Introduction

1. Antenna: structures designed for radiating electromagnetic energy efficiently in a prescribed manner.

2. EM energy is radiated from the time-varying charge and current distributions in the antennas.

3. To study radiation from charge or current distribution, non-homogeneous wave equations for potentials V and $\mathbf{A}$ will be used. The following is a brief review and introduction of the potential functions. (see §7-4 and §7-6)

4. Potential functions for static fields

$$\begin{cases} \nabla \times \mathbf{E} = 0 \\ \nabla \cdot \mathbf{B} = 0 \end{cases} \Rightarrow \begin{cases} \mathbf{E} = -\nabla V, & V = \\ \\ \mathbf{B} = \nabla \times \mathbf{A}, & \mathbf{A} = \end{cases}$$

5. Potential functions for time-varying fields

6. Definition of potential function $\mathbf{A}$

   From $\nabla \times \mathbf{H} = \mathbf{J} + \dfrac{\partial \mathbf{D}}{\partial t}$,

   $\Rightarrow$

   For homogeneous linear medium,

   $$\nabla \times \nabla \times \mathbf{A} = \mu \mathbf{J} + \mu\epsilon \frac{\partial}{\partial t}\left(-\nabla \mathbf{A} - \frac{\partial \mathbf{A}}{\partial t}\right) = \nabla(\nabla \cdot \mathbf{A}) - \nabla^2 \mathbf{A}$$

   $\Rightarrow$

$\Rightarrow$ **A** can be defined such that

$$\Rightarrow \nabla^2 \mathbf{A} - \mu\epsilon \frac{\partial^2 \mathbf{A}}{\partial t^2} = -\mu \mathbf{J} \qquad \text{(7-63)}$$

(Non-homogeneous wave equation for **A**)

Similarly from the Lorentz's condition,

$$\Rightarrow \nabla^2 V - \mu\epsilon \frac{\partial^2 V}{\partial t^2} = -\frac{\rho}{\epsilon}$$

(Non-homogeneous wave equation for V)

- **A** and V (**J** and ρ) are not independent. They are related by Lorentz's condition (continuity equation).
- The Lorentz's condition uncouples the wave equations for **A** and V.
- The non-homogeneous wave equations reduce to Poisson's equations in static cases.

7. Wave equations for potential functions (§7-6)

   For given time-varying charge ρ and current **J** distributions, V and **A** can be solved form the non-homogeneous potential wave equations and **E** and **B** can be determined. For the scalar potential V in a static case, the wave equation reduces to the Poisson's equation, $\nabla^2 V = -\rho/\epsilon$, and the solution is

   In a time varying case, it can be shown that the time varying potential has the solution

$$V(\mathbf{R}, t) = \frac{1}{4\pi\epsilon} \int_{V'} \frac{\rho(\mathbf{R}', t - \frac{|\mathbf{R} - \mathbf{R}'|}{u}) dv'}{|\mathbf{R} - \mathbf{R}'|}, \text{ where } u = \frac{1}{\mu\epsilon}$$

- The potential V at **R** at time t is equal to the sum of Coulomb potentials of dq' at $\mathbf{R}'$ at an earlier time $t - |\mathbf{R} - \mathbf{R}'|/u$

$\Rightarrow$ the effect of dq' takes time to travel to $\mathbf{R}$ with speed $u = 1/\sqrt{\mu\epsilon}$

$\Rightarrow$ $V(\mathbf{R}, t)$ is also called retarded potential.

Similarly for $\mathbf{A}$,

$$\mathbf{A}(\mathbf{R}, t) = \frac{\mu}{4\pi} \int_{V'} \frac{\mathbf{J}(\mathbf{R}', t - |\mathbf{R} - \mathbf{R}'|/u)}{|\mathbf{R} - \mathbf{R}'|} \; dv'$$

If $|\mathbf{R} - \mathbf{R}'|/u \ll t$ (slow time variation or short distance),

$$V(\mathbf{R}, t) \approx \frac{1}{4\pi\epsilon} \int_{V'} \frac{\rho(\mathbf{R}', t)}{|\mathbf{R} - \mathbf{R}'|} dv'$$

- quasi static approximation
- encountered in low frequency circuitry

8. From the potentials, $\mathbf{E}$ and $\mathbf{H}$ fields can be calculated,

$$\begin{cases} \mathbf{H} = \dfrac{\mathbf{B}}{\mu} = \dfrac{1}{\mu} \nabla \times \mathbf{A} & (11-1) \\ \mathbf{E} = -\nabla V - \dfrac{\partial \mathbf{A}}{\partial t} & (11-2) \end{cases}$$

9. In time harmonic fields, the phasor form of the retarded potential can be found by

$\mathbf{A}(\mathbf{R}, t) = \Re[\mathbf{A}(\mathbf{R}) e^{j\omega t}] =$

In phasor form,

- The radiation pattern can be calculated by:

  $\mathbf{J} \rightarrow \mathbf{A} \rightarrow \mathbf{H}$ and $\rho \rightarrow V \rightarrow \mathbf{E}$ (with knowledge of $\mathbf{A}$)

- $\mathbf{E}$ field can also be calculated from the $\mathbf{H}$ field using Maxwell's equation, $\mathbf{E} = \dfrac{1}{j\omega\epsilon} \nabla \times \mathbf{H}$

11.2 Radiation Fields of Dipoles

11.2.1 Electrical Dipoles (Hertzian Dipole)

1. Assume a uniform harmonic current

   The electrical dipole moment in phasor form is

   Vector potential due to **p** is

2. In spherical coordinate,

   $\mathbf{H} = \frac{1}{\mu_0}\nabla \times \mathbf{A} = \mathbf{a}_\phi \frac{1}{\mu_0 R}\left[\frac{\partial}{\partial R}(RA_\theta) - \frac{\partial A_R}{\partial \theta}\right]$

   $=$

   $\Rightarrow \mathbf{E} = \frac{1}{j\omega\epsilon_0}\nabla \times \mathbf{H} = \mathbf{a}_R E_R + \mathbf{a}_\theta E_\theta + \mathbf{a}_\phi E_\phi$

   where

$$\begin{cases} E_R &= -\frac{Id\ell}{4\pi}\,\eta_0\,\beta^2\,2\cos\theta\left[\frac{1}{(j\beta R)^2} + \frac{1}{(j\beta R)^3}\right]e^{-j\beta R} \\ E_\theta &= -\frac{Id\ell}{4\pi}\,\eta_0\,\beta^2\,\sin\theta\left[\frac{1}{j\beta R} + \frac{1}{(j\beta R)^2} + \frac{1}{(j\beta R)^3}\right]e^{-j\beta R} \\ E_\phi &= 0 \end{cases}$$

(a) Near field: $\beta R = 2\pi \frac{R}{\lambda} \ll 1$

$\Rightarrow$ near field = quasi static field, no phase delay

(b) Far field: $\beta R = 2\pi \frac{R}{\lambda} \gg 1 \Rightarrow$ keep only $1/j\beta R$ terms

i. E_θ, H_ϕ in time phase, $\mathbf{E} \perp \mathbf{H}$

ii. $E_\theta / H_\phi = \eta_0 =$ intrinsic impedance of the medium

iii. $|E_\theta|, |H_\phi| \propto 1/R, |\mathrm{P}_{\mathrm{av}}| \propto 1/R^2$, spherical wave

11.2.2 Magnetic Dipoles

Radiation can also be generated by a magnetic dipole with the far fields

$$\begin{cases} E_\phi = \frac{\omega \mu_0 m}{4\pi} \left(\frac{e^{-j\beta R}}{R} \right) \beta \sin\theta \\ H_\theta = -\frac{\omega \mu_0 m}{4\pi \eta_0} \left(\frac{e^{-j\beta R}}{R} \right) \beta \sin\theta \quad \Rightarrow \text{spherical waves} \\ H_R = 0 \end{cases}$$

11.3 Antenna Pattern (Radiation Pattern)

1. To describe the far field distribution in all 3-D directions (constant R),

   E-plane pattern: normalized field strength vs. θ for constant ϕ

   H-plane pattern: normalized field strength vs. ϕ for $\theta = \pi/2$

Ex 11-1: Radiation pattern of Hertzian dipoles

$$\begin{cases} H_\phi = j\dfrac{Id\ell}{4\pi}\dfrac{e^{-j\beta R}}{R}\,\beta\,\sin\theta \\ E_\theta = j\dfrac{Id\ell}{4\pi}\dfrac{e^{-j\beta R}}{R}\,\eta_0\,\beta\,\sin\theta \end{cases}$$

(a) E-plane pattern

(b) H-plane pattern

2. Typical radiation patterns and parameters (Fig 11-4)

(a) Width of main beam (beamwidth)

$$= \text{angular width between} \begin{cases} \text{half power (3dB) points (field strength} = 1/\sqrt{2}) \\ \text{-10 dB points} \\ \text{first nulls} \end{cases}$$

(b) Sidelobe levels: as low as possible for directive pattern ($\sim$ -40 dB)

(c) Directivity: ability to radiate power in a given direction

Consider time-averaged total radiated power

where $\mathbf{P}_{\text{av}}$=Poynting vector,

$d\Omega$ = differential solid angle = $\sin\theta\, d\theta\, d\phi = ds/R^2$,

U = radiation intensity = power per unit solid angle = $R^2\mathbf{P}_{\text{av}}$

Directive gain $G_D(\theta,\phi)$ is defined as

$$G_D(\theta,\phi) = \frac{P_{\text{av}}}{P_r/4\pi R^2} = \frac{U(\theta,\phi)}{P_r/4\pi}$$

Directivity is defined as

(d) Efficiency: P_r is the radiated power in the far field. The total input electric power to the antenna P_i is composed of the radiated power P_r and loss power P_ℓ due to the antenna itself and nearby lossy structures (including ground). The radiation efficiency η_r is defined as

(Normally, the efficiency of a well-constructed antenna is close to 100%.)

(e) Radiation resistance: With input current I to the antenna, the radiation resistance R_r of the antenna is the equivalent resistance that would dissipate an amount of power equal to the radiated power P_r when the same current I passes through P_r.

Ex 11-2, 11-3: Hertzian diople

$$\begin{aligned}\mathbf{P}_{\text{av}} &= \frac{1}{2}\Re[\mathbf{E}\times\mathbf{H}^*] \\ &= \frac{1}{2}|E_\theta||H_\phi|\end{aligned}$$

- short dipole antenna is a poor radiator (small R_r)
- short dipole antenna has a large capacitive reactance $\Rightarrow$ difficult to match and couple energy

11.4 Linear Dipole Antenna

1. "Assume" the current in the antenna is

   In the far field, the differential E-field intensity due to a differential current element Idz is

   $dE_\theta = \eta_0 dH_\phi = j\frac{Idz}{4\pi}\left(\frac{e^{-j\beta R'}}{R'}\right)\eta_0\beta\sin\theta$

   $\Rightarrow E_\theta = \eta_0 H_\phi = \int_{-h}^{h} dE_\theta = \int_{-h}^{h} j\frac{I(z)}{4\pi}\frac{e^{-j\beta R'(z)}}{R'(z)}\eta_0\beta\sin\theta dz$

   where $F(\theta) = \dfrac{\cos(\beta h\cos\theta) - \cos\beta h}{\sin\theta}$ = pattern function

11.4.1 Half wave dipole

1. Far field: $E_\theta = \dfrac{j\,60 I_m}{R}\, e^{-j\beta R} \left(\dfrac{\cos\left[\dfrac{\pi}{2}\cos\theta\right]}{\sin\theta} \right)$

$$H_\phi = \frac{j\,I_m}{2\pi R}\, e^{-j\beta R} \left(\frac{\cos\left[\frac{\pi}{2}\cos\theta\right]}{\sin\theta} \right)$$

2. Poynting vector

3. Total radiated power

Ex 11-5: Quarter wave monopole