10 Waveguide

10.1 Introduction (also see §9.1)

1. EM wave in free space: unguided, less efficient
2. EM waves can be guided by waveguides or transmission lines for more efficient power transmission

10.2 Uniform Guiding Structures

1. Straight guiding structure with uniform cross section

   In phasor form,

   The phasor wave equations (Helmholtz's equation) are,

   $$\begin{cases} \nabla^2\mathbf{E} + k^2\mathbf{E} = 0 \\ \nabla^2\mathbf{H} + k^2\mathbf{H} = 0 \end{cases}, \quad k = \omega\sqrt{\mu\epsilon} = 2\pi/\lambda$$

   The Laplacian operator ∇ can be written as
   $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} =$

   $$\Rightarrow \begin{cases} \nabla^2\mathbf{E} + k^2\mathbf{E} = \\ \nabla^2\mathbf{H} + k^2\mathbf{H} = \nabla^2_{xy}\mathbf{H} + (\gamma^2 + k^2)\mathbf{H} = 0 \end{cases}$$

   where $\begin{cases} k = \text{wave nubmer of plane wave along any direction in free sapce,} \\ \gamma = \text{wave number of guided wave in waveguides along } z. \end{cases}$

2. With $\partial/\partial z \to -\gamma$, the six differential equations for the field components can be obtained from Maxwell's equations,

$\nabla \times \mathbf{E} = -j\omega\mu\mathbf{H}$ $\nabla \times \mathbf{H} = j\omega\epsilon\mathbf{E}$

$$\Rightarrow \begin{cases} \dfrac{\partial E_z^0}{\partial y} + \gamma E_y^0 &= -j\omega\mu H_x^0 \\ -\gamma E_x^0 - \dfrac{\partial E_z^0}{\partial x} &= -j\omega\mu H_y^0 \\ \dfrac{\partial E_y^0}{\partial x} - \dfrac{\partial E_x^0}{\partial y} &= -j\omega\mu H_z^0 \end{cases} \qquad \Rightarrow \begin{cases} \dfrac{\partial H_z^0}{\partial y} + \gamma H_y^0 &= j\omega\epsilon E_x^0 \\ -\gamma H_x^0 - \dfrac{\partial H_z^0}{\partial x} &= j\omega\epsilon E_y^0 \\ \dfrac{\partial H_y^0}{\partial x} - \dfrac{\partial H_x^0}{\partial y} &= j\omega\epsilon E_z^0 \end{cases}$$

The field components are not all independent. Usually, the transvers field components (x, y) are expressed in terms of the longitudinal components (z):

$$\Rightarrow \begin{cases} H_x^0 &= -\dfrac{1}{h^2}\left(\gamma\dfrac{\partial H_z^0}{\partial x} - j\omega\epsilon\dfrac{\partial E_z^0}{\partial y}\right) & (10-11) \\ H_y^0 &= -\dfrac{1}{h^2}\left(\gamma\dfrac{\partial H_z^0}{\partial y} + j\omega\epsilon\dfrac{\partial E_z^0}{\partial x}\right) & (10-12) \\ E_x^0 &= -\dfrac{1}{h^2}\left(\gamma\dfrac{\partial E_z^0}{\partial x} + j\omega\mu\dfrac{\partial H_z^0}{\partial y}\right) & (10-13) \\ E_y^0 &= -\dfrac{1}{h^2}\left(\gamma\dfrac{\partial E_z^0}{\partial y} - j\omega\mu\dfrac{\partial H_z^0}{\partial x}\right) & (10-14) \end{cases}$$

where $h^2 = k^2 + \gamma^2$.

$\Rightarrow E_z^0$ and/or H_z^0 are first solved from the wave equations and boundary condtions, and $E_x^0, E_y^0, H_x^0, H_y^0$ can be obtained from the above equaiotns (Eqs. 10-11 $\sim$ 10-14).

- TEM waves: $E_z^0 = 0, H_z^0 = 0$, no longitudinal field components
- TM waves: $E_z^0 \neq 0, H_z^0 = 0$, no longitudinal **H** field components
- TE waves: $E_z^0 = 0, H_z^0 \neq 0$, no longitudinal **E** field components

10.3 Parallel-Plate Waveguide

- neglect fringing effect ($W \gg b$)
- perfect conductor
- lossless dielectric

1. TEM wave (see §9-2)

   Consider a y-polarized uniform plane wave propagating in the $+z$ direction between the two conductors

   Boundary conditions at $y = 0$ and $y = b$: $E_t = 0$ and $H_n = 0$

   $\Rightarrow$ satisfied because of no tangential E and normal H in the above wave

   $\Rightarrow$ The plane wave is a solution (mode) in the parallel plate waveguide

   - The propagation characteristics (γ, η, u_p) of a guided TEM mode are similar to those of a plane wave in free space.
   - In terms of Eq. 10-2, $\mathbf{E}^0(x, y) =$ constant.

2. Surface charge and current due to discontinuity of E_n and H_t:

   On lower plate ($y = 0$), $\mathbf{a}_n = \mathbf{a}_y$, from bonudary conditions,

   On upper plate ($y = b$), $\mathbf{a}_n = -\mathbf{a}_y$,

$$\begin{cases} -\mathbf{a}_y \cdot \mathbf{D} = \rho_{su} \quad \Rightarrow \quad \rho_{su} = -\epsilon E_y = -\epsilon E_0 e^{-j\beta z} \\ -\mathbf{a}_y \times \mathbf{H} = \mathbf{J}_{su} \quad \Rightarrow \quad \mathbf{J}_{su} = \mathbf{a}_z H_x = -\mathbf{a}_z (E_0/\eta) e^{-j\beta z} \end{cases}$$

10.3.1 TM waves ($H_z = 0, E_z \neq 0$)

1. To find a TM wave (mode) in the waveguide,

(h = eigen value; $E_z^0(y)$ = eigen function/mode/vector)

B.C.: $E_z^0(y) = 0$ at $y = 0$ and $y = b$ ($E_t = 0$)

$\Rightarrow$

$$\Rightarrow \begin{cases} H_x^0(y) = \dfrac{j\omega\epsilon}{h^2}\dfrac{\partial E_z^0}{\partial y} = \dfrac{j\omega\epsilon}{h} A_n \cos\dfrac{n\pi y}{b} & \text{(from Eq. 10-11)} \\ H_y^0(y) = -\dfrac{j\omega\epsilon}{h^2}\dfrac{\partial E_z^0}{\partial x} = 0 & \text{(from Eq. 10-12)} \\ E_x^0(y) = -\dfrac{\gamma}{h^2}\dfrac{\partial E_z^0}{\partial x} = 0 & \text{(from Eq. 10-13)} \\ E_y^0(y) = -\dfrac{\gamma}{h^2}\dfrac{\partial E_z^0}{\partial y} = -\dfrac{\gamma}{h} A_n \cos\dfrac{n\pi y}{b} & \text{(from Eq. 10-14)} \end{cases}$$

where $\gamma^2 = h^2 - k^2 = \left(\frac{n\pi}{b}\right)^2 - \omega^2\mu\epsilon$.

$\Rightarrow \gamma =$

(a) $f > f_c, \gamma = j\beta \Rightarrow$ propagation mode

(b) $f < f_c, \gamma = \alpha \Rightarrow$ decay/evanescent mode

Ex 10-3: TM_1 mode ($n = 1$)

(a) instantaneous expressions

$$\begin{cases} E_z(y, z, t) = \\ \\ E_y(y, z, t) = \cdots = \dfrac{\beta b}{\pi} A_1 \cos\left(\dfrac{\pi y}{b}\right) \sin(\omega t - \beta z) \\ H_x(y, z, t) = \cdots = -\dfrac{\omega\epsilon b}{\pi} A_1 \cos\left(\dfrac{\pi y}{b}\right) \sin(\omega t - \beta z) \end{cases}$$

(b) field lines

E field lines in $y - z$ plane:

H field lines:

$\mathbf{H} = \mathbf{H}_x =$ perpendicular to yz plan

(J_s can be found at $y = 0, b$)

Ex 10-4: TM_1 mode as superposition of two plane waves

$\Rightarrow \mathbf{k}_1, \mathbf{k}_2$ bouncing up and down, no net z propagation

2. At cutoff ($\lambda/2b = 1$)

   $\rightarrow$ time harmonic field with no spatial variation in z direction

   - In the waveguide, $\mathbf{k}_1, \mathbf{k}_2$ form standing wave that satisfies the B.C..
   - For $\lambda > \lambda_c, f < f_c$, standing waves satisfying the B.C. can not be formed.

10.3.2 TE wave ($E_z = 0; H_z \neq 0$)

1. $E_z = 0$, from Eq. 10-48,

   B.C.: $E_x = 0$ at conductor surfaces

$$\Rightarrow \begin{cases} H_y^0(y) &= \frac{\gamma}{h} B_n \sin \frac{n\pi y}{b} \\ E_x^0(y) &= \frac{j\omega\mu}{h} B_n \sin \frac{n\pi y}{b} \end{cases}$$

   where $\gamma = \sqrt{\left(\frac{n\pi}{b}\right)^2 - \omega^2\mu\epsilon}$.

   - $f_{c,\mathrm{TE}} = f_{c,\mathrm{TM}} = n/2b\sqrt{\mu\epsilon}$
   - For $n = 0, H_y = 0, E_x = 0$

     $\Rightarrow$ TE_0 mode does not exist in parallel plate waveguides

Ex 10-5: (see textbook) (Fig. 10-8)

10.6 Dielectric waveguide

10.6.1 TM mode in a slab dielectric waveguide

1. For TM modes, $H_z = 0$

   EM fields exist in both free space and dielectric slab. Waveguide modes can be found by matching the B.C. for wave solutions of Eq. 10-241 in the three regions.

   (a) In the slab, $|y| \le d/2$,

   $\Rightarrow$ solution of Eq. 10-241 is

   (b) In free space, $y \ge d/2$ and $y \le -d/2$, the solution of Eq. 10-241 can be sinusoidal or $\pm$ exponential functions. But waveguide modes must be confined to the region near the slab.

   $\Rightarrow$

(c) k_y, α: to be determined by B.C.

E_o, E_e, C_u, C_ℓ: to be determined by initial conditions

i. odd TM mode

$E_z^0(y)$ is described by a sine function and is antisymmetric with respect to the $y = 0$ plane.

A. in the dielectric region, $|y| \leq d/2$, from Eqs. 10-244, 10-11, 10-14,

$$\begin{cases} E_z^0(y) &= \\ \\ E_y^0(y) &= -\dfrac{j\beta}{k_y} E_0 \cos k_y y \\ H_x^0(y) &= \dfrac{j\omega\epsilon d}{k_y} E_0 \cos k_y y \end{cases}$$

B. in the upper free space, $y \geq d/2$

$$\begin{cases} E_z^0(y) &= \\ \\ E_y^0(y) &= -\dfrac{j\beta}{\alpha} E_0 \sin \dfrac{k_y d}{2} e^{-\alpha(y-d/2)} \\ H_x^0(y) &= \dfrac{j\omega\epsilon_0}{\alpha} E_0 \sin \dfrac{k_y d}{2} e^{-\alpha(y-d/2)} \end{cases}$$

($C_u = E_0 \sin \frac{k_0 d}{2}$ from B.C.)

C. in the lower free space, $y \leq d/2$

$$\begin{cases} E_z^0(y) &= \\ \\ E_y^0(y) &= -\dfrac{j\beta}{\alpha} E_0 \sin \dfrac{k_y d}{2} e^{\alpha(y+d/2)} \\ H_x^0(y) &= \dfrac{j\omega\epsilon_0}{\alpha} E_0 \sin \dfrac{k_y d}{2} e^{\alpha(y+d/2)} \end{cases}$$

D. From B.C., tangential H (H_x) must be continuous at $y = \pm d/2$,

$\Rightarrow$ From Eqs. 10-250, 10-253,

$\Rightarrow$ Eq. 10-260 can be solved numerically or graphically

ii. Similarly, even TM modes can be obtained from

note:
- must have $\mu_d\epsilon_d > \mu_0\epsilon_0$ (or $\epsilon_d > \epsilon_0$ for non-magnetic materials) to have guided modes
- guided modes are "discrete"
- when ω increases, more modes can be supported
- when d increases, more modes can be supported
- when $\mu_d\epsilon_d - \mu_0\epsilon_0$ increases, more modes can be supported
- optical fiber is a cylindrical dielectric waveguide