§8-5 Probability

＊Discrete Random Variables：例如擲骰子 $\Rightarrow$ $X=\{1,2,3,4,5,6\}$

Continuous Random Variables：例如國人的平均身高 $X=\{0\le x\le 250\}$

＊離散：

- 若公正的骰子，則 $p(x=1)=\frac{1}{6}=p(x=2)$
- 離散的 probability density function

  $f: X=\{1,2,3,4,5,6\}\to R^{+}\cup\{0\}$

  $f(1)=f(2)=f(3)=f(4)=f(5)=f(6)=\frac{1}{6}.$

＊連續：

- 國人的身高 $X=\{0\le x\le 250\}$，在 0 公分和 250 公分之間的實數有無窮多個，因此算單一身高(例如 170.123 公分)的機率是無意義的，也是不必要的，所以在考慮連續型的 random variables 和離散型 random variables 的數學處理方法有很大的差別，在連續型我們是先猜測或判斷這個 random variables 的 probability density function(pdf) 的可能形狀。

  1. exponentially decreasing pdf：常用在等時間或產品出問題的時間上。
  2. 常態分佈的 pdf。
  3. 通常 pdf 的定義域可以寫為 **R**，在不可能出現的範圍，函數值可設為零。

- 給定一個 continuous random variable 的 pdf，則一些相關的機率問題和離散型很像，這兩者關係就如 $\int$(連續的加法符號)和 Σ(離散的加法符號)的關係一樣。

Given X, f (pdf). Then：

- $p(a\le x\le b)=\int_a^b f(x)dx$ (和離散的觀念是一致的)
- 在擲公正骰子的例子中：

$p(2 \le x \le 4) = f(2) + f(3) + f(4) = \frac{1}{6} + \frac{1}{6} + \frac{1}{6} = \frac{1}{2}.$

平均值(期望值): $\sum_{k=1}^{6} kf(k) = \mu$

Variance(變異數, 方差): $\sum_{k=1}^{6} (k - \mu)^2 f(k) = Var(X)$

Given X(連續), f. Then

The mean(μ 平均值) of f is defined by

$$\mu = \int_{-\infty}^{\infty} xf(x)dx$$
$$Var(X) = \int_{-\infty}^{\infty} (x - \mu)^2 f(x)dx$$

Example 1：

Suppose a problem such as waiting time, equipment failure times can be modeled by a pdf which is exponentially decreasing. Find such pdf.

Example 2：

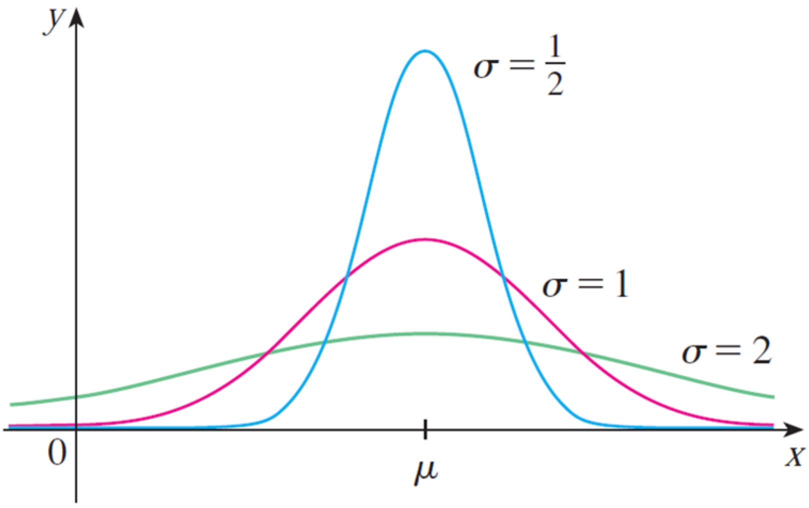
Suppose the average waiting time for a customer's call to be answered by a company representative is five minutes.

(a) Find the probability that a call is answered during the first minutes.

(b) Find the probability that a customer waits more than five minutes to be answered.

＊pdf of Normal Distributions：

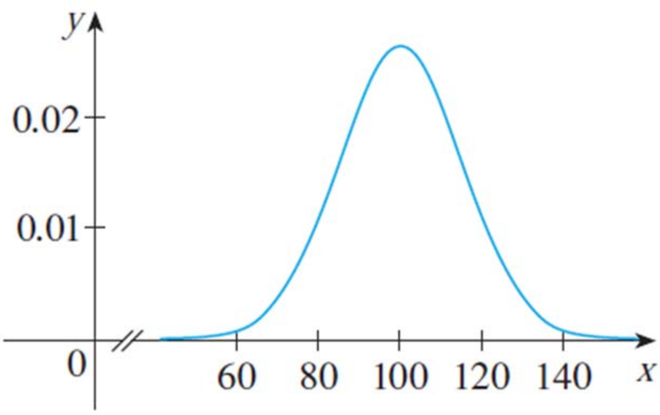
例子：Test scores, heights, weight, annual rainfall…



$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(x-\mu)^2}{2\sigma^2}}$$

$$\int_{-\infty}^{\infty} e^{-\frac{(x-\mu)^2}{2\sigma^2}} = \sqrt{2\pi}\sigma; \int_{-\infty}^{\infty} xf(x) = \mu; \int_{-\infty}^{\infty} (x-\mu)^2 f(x) = \sigma$$

Example 3：Intelligence Quotient (IQ) scores are distributed normally with mean 100 and standard deviation 15. (Figure shows the corresponding probability density function)



(a) What percentage of the population has an IQ score between 85 and 115?

(b) What percentage of the population has an IQ above 140?