

§6-1 Areas Between Curves

* Keys :

- Σ (離散的加法符號)

\int (連續的加法符號)

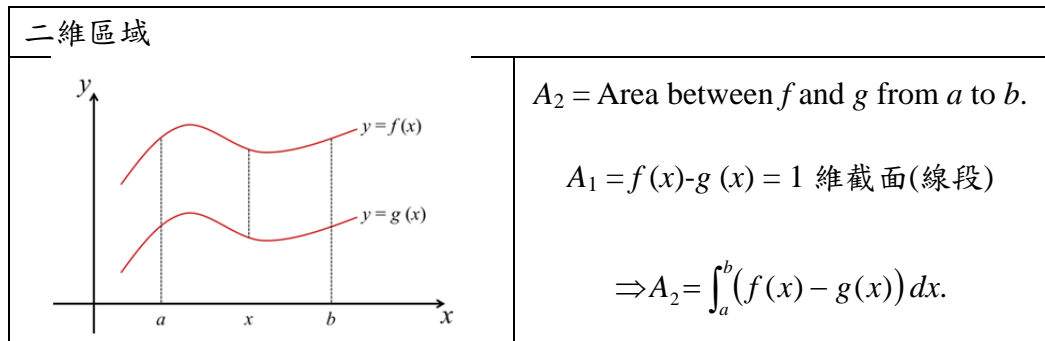
- $A_n = N$ 維物體的體積

A_{n-1} = 此物體 n-1 維截面的體積

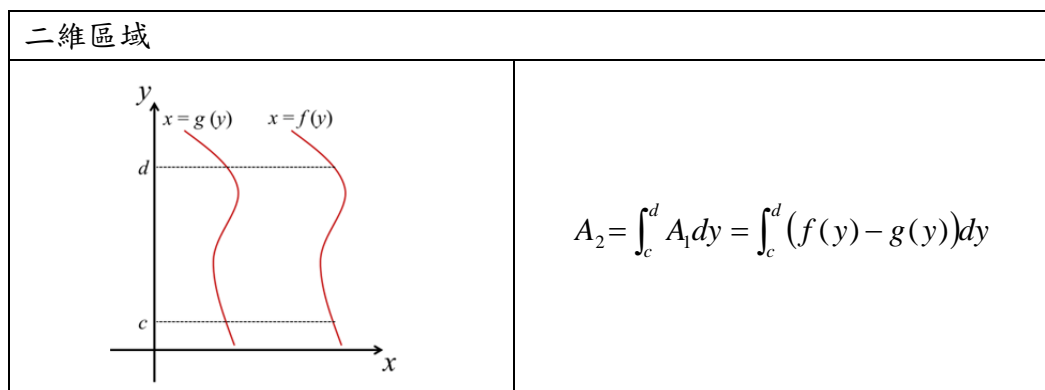
$$\Rightarrow A_n = \int A_{n-1} dx = \int (\text{截面})_{n-1} dx$$

Examples :

(a)

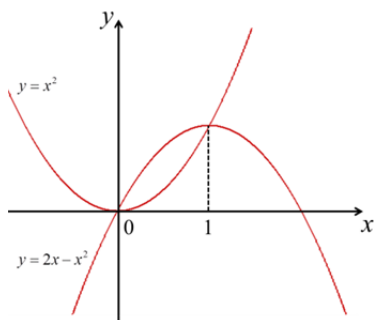


(b)



Example 1 :

Find the area of the region enclosed by the parabolas $y = x^2$ & $y = 2x - x^2$.

Solution :

$$\begin{cases} y = x^2 \\ y = 2x - x^2 \end{cases}$$

$$\Rightarrow x^2 = 2x - x^2$$

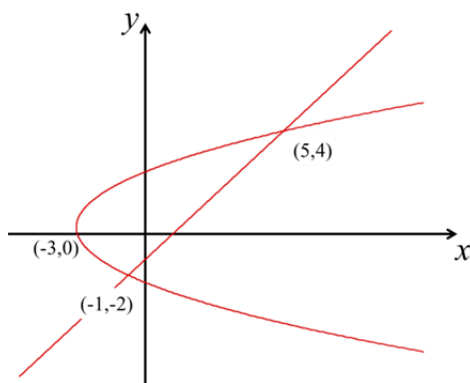
$$\Rightarrow x = 0, 1$$

$$A = \int_0^1 (2x - x^2 - x^2) dx = 2 \int_0^1 (x - x^2) dx$$

$$= 2 \left(\frac{x}{2} - \frac{x^3}{3} \right)_0^1 = \frac{1}{3}.$$

Example 2 :

Find the area enclosed by the line $y = x - 1$ and the parabolas $y^2 = 2x + 6$.

Solution :

$$\begin{cases} y = x - 1 \\ y^2 = 2x + 6 \end{cases}$$

$$\Rightarrow (x - 1)^2 = 2x + 6$$

$$\Rightarrow x = 5, -1$$

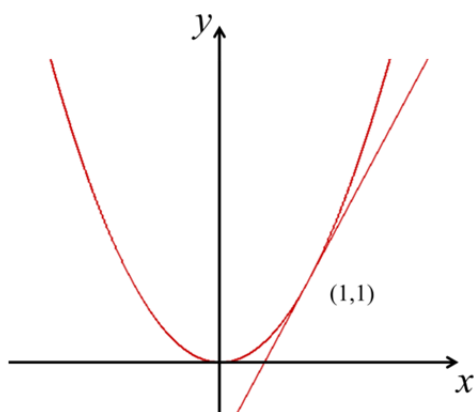
$$\Rightarrow y = 4, -2$$

$$A = \int_{-3}^{-1} 2\sqrt{2x+6} dx + \int_{-1}^5 [\sqrt{2x+6} - (x+1)] dx$$

$$= \int_{-2}^4 \left[y + 1 - \frac{y^2 - 6}{2} \right] dy = 18.$$

Example 3 : Find the area of the region bounded by $y = x^2$, the tangent line to this parabola at (1,1), and the x -axis.

Solution :



$$\begin{aligned} y &= x^2 \\ \Rightarrow y' &= 2x \\ \Rightarrow y'(1) &= 2 \end{aligned}$$

Tangent line at (1,1) :

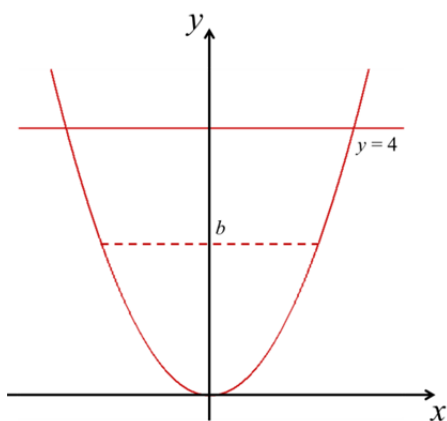
$$y - 1 = 2(x - 1)$$

$$\Rightarrow x = \frac{y+1}{2}$$

$$A = \int_0^1 \left(\frac{y+1}{2} - \sqrt{y} \right) dy = \frac{1}{12}.$$

Example 4 : Find the number b such that the line $y = b$ divides the region bounded by the curve $y = x^2$ and $y = 4$ into two region with equal area.

Solution :



$$\frac{1}{2} \int_0^2 (4 - x^2) dx = \int_0^{\sqrt{b}} (b - x^2) dx$$

$$\Rightarrow \frac{8}{3} = \frac{2}{3} b^{\frac{3}{2}}$$

$$\Rightarrow b = 4^{\frac{2}{3}}.$$