

§5-4 Indefinite Integrals

* Indefinite Integral :

$\int f(x)dx =$ The most general form of the antiderivatives of f .

$= F(x) + c$, where $F'(x) = f(x)$.

Examples :

(i) $\int x^n dx = \frac{x^{n+1}}{n+1} + C.$	(ii) $\int e^x dx = e^x + C.$
(iii) $\int \sin x dx = -\cos x + C.$	(iv) $\int \frac{1}{x} dx = \ln x + C.$
(v) $\int \frac{1}{1+x^2} dx = \tan^{-1} x + C.$	(vi) $\int \frac{1}{\sqrt{1-x^2}} dx = \sin^{-1} x + C.$

Examples 1 : $\int \frac{2t^2 + t^2\sqrt{t} - 1}{t^2} dx = ?$

Solution :

$$\int \frac{2t^2 + t^2\sqrt{t} - 1}{t^2} dx = \int (2 + \sqrt{t} - t^{-2}) dx = 2t + \frac{2}{3}t^{\frac{3}{2}} + t^{-1} + C.$$

Example 2 : $\int (10x^4 - \sec^2 x) dx = ?$

Solution :

$$\int (10x^4 - \sec^2 x) dx = 2x^5 - \tan x + C.$$

Example 3 : $\int_{-1}^2 (x - 2 |x|) dx = ?$

Solution :

$$\begin{aligned}\int_{-1}^2 (x - 2 |x|) dx &= \int_{-1}^0 (x + 2x) dx + \int_0^2 (x - 2x) dx \\ &= \frac{3}{2} x^2 \Big|_{-1}^0 - \frac{1}{2} x^2 \Big|_0^2 = -\frac{3}{2} - 2 = -\frac{7}{2}.\end{aligned}$$