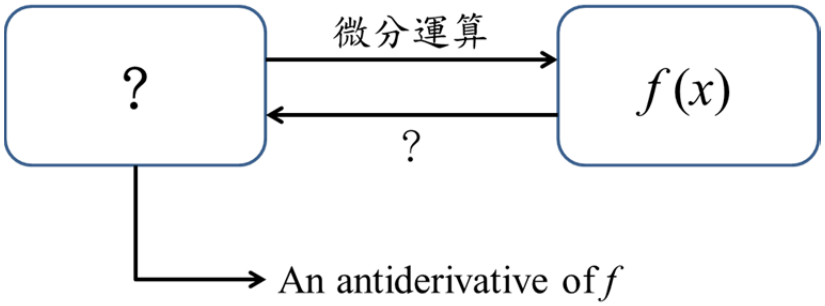
§4-9 Antiderivatives



＊Definition：

If $F'(x) = f(x)$ for all $x \in I$, then F is called an antiderivative of f on I.

Example 1：Find two antiderivatives of $f(x) = x$.

Solution：

$$F(x) = \frac{x^2}{2} + 1, \quad F(x) = \frac{x^2}{2} - 1$$

＊Theorem：

If F is an antiderivative of f on I, then the most general antiderivative of f on I is $F(x) + C$, where C is an arbitrary constant.

Example 2： $f'(u) = \dfrac{u^4 + 3\sqrt{u}}{u^2}$, find $f(u)$.

Solution：

$$f'(u) = u^2 + 3u^{-\frac{3}{2}} \Rightarrow f(u) = \frac{u^3}{3} - 6u^{-\frac{1}{2}} + C$$

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Example 3： $f'(x)=\sqrt{x}(6+5x),\quad f(1)=10$, find $f(x)$.

Solution：

$$f'(x)=6x^{\frac{1}{2}}+5x^{\frac{3}{2}}$$

$$\Rightarrow f(x)=4x^{\frac{3}{2}}+2x^{\frac{5}{2}}+C$$

$$\Rightarrow f(1)=4+2+C=10\Rightarrow C=4$$

$$\Rightarrow f(x)=4x^{\frac{3}{2}}+2x^{\frac{5}{2}}+4.$$

Example 4：What constant acceleration is required to increase the speed of a car from 50 *km*/*h* to 80 *km*/*h* in 5*s* ?

Solution：

Let $a(t)=k$ be the constant acceleration.

$$\Rightarrow V(t)=kt+C$$

$$V(0)=C=50,\quad V(5)=5k+C=80$$

$$\Rightarrow k=6,\quad C=50$$

$$\Rightarrow a(t)=k=6.$$