

§10-1 Curves Defined by Parametric Equation

* Parameter Equation : $x = f(t), y = g(t)$.

Parameter Curve : $\{(x, y) : x = f(t), y = g(t)\}$.

Example 1 :

Verify the curve $x = \cos t, y = \sin t, 0 \leq t \leq 2\pi$.

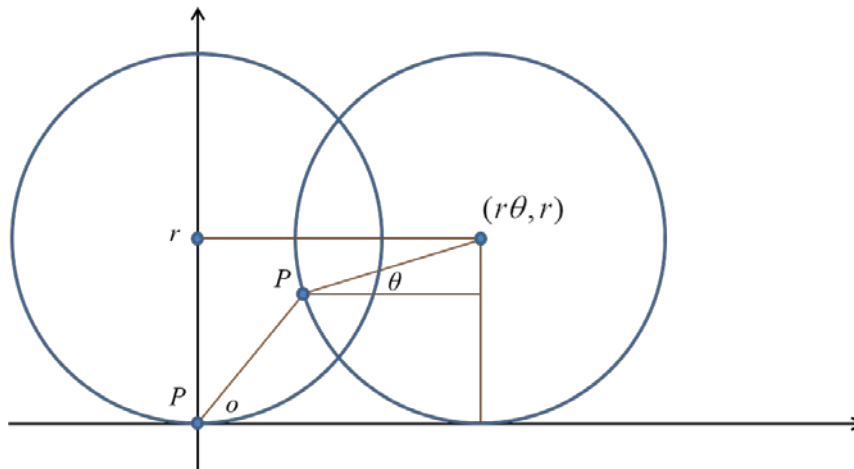
Solution :

The circle : $x^2 + y^2 = 1$

Example 2 :

The curve traced out by a point P on the circumference of a circle as the circle rolls along a straight line is called a cycloid. Find parametric equation for the cycloid.

Solution :



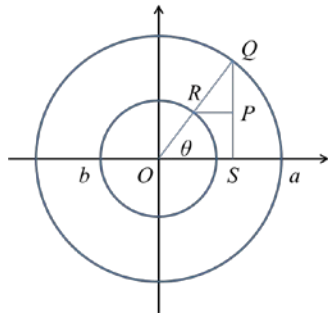
$p(x, y)$

$$x = r\theta - r \sin \theta$$

$$y = r - r \cos \theta.$$

Example 3 :

Find $P(x,y)$ in terms of Q . What is its parametric curve?



Solution :

$$x = \overline{SO} = a \cos \theta, \quad y = \overline{SP} = b \sin \theta$$

$$\Rightarrow \cos \theta = \frac{x}{a} \text{ and } \sin \theta = \frac{y}{b}$$

$$\Rightarrow \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1.$$